Algebra 2 Name: ______________________

Section 5.5 – Notes and Examples Date: ______________ Hour: ______

Theorems About Roots of Polynomial Equations

real
- A *rational* number is ratio of 2 integer   1/2, 1/3, 2/5, .75 .
- An *irrational* number includes values like: $\pm\sqrt{2}$ $3\pm\sqrt{5}$, $7\pm\sqrt{7}$

When you solve polynomial equations, some solutions might be rational, while others can be irrational. **If a polynomial is factorable, you will get rational solutions**. If a problem has a square root in its solution, that is considered irrational.

But not all polynomials are factorable! The ***Rational Root Theorem*** (also known as the Rational Zero Theorem) can help you find all **possible** rational roots of a polynomial equation.

Rational Root Theorem

If the polynomial $P(x)$ has integer coefficients, then every rational root of the polynomial equation $P(x) = 0$ can be written in the form $\frac{p}{q}$, where p is a factor of the constant term of $P(x)$ and q is a factor of the leading coefficient of $P(x)$.

Follow these steps to find the list of possible rational zeros:
- Write the equation in standard form, looking for missing terms.
- List all factors of the constant term. This is often called "p."
- List all factors of the leading coefficient. This is often called "q."
- List all combinations of fractions created from $\frac{p}{q}$, including positives and negatives.
- Test each possibility using synthetic divison until you find one with a zero remainder. Always try 1 and –1 first!
- Use the quadratic formula or factor to find the remaining roots.

The Quadratic Formula often is a better choice than factoring, since not all of the polynomials will be factorable!

Use the Rational Root Theorem (RRT) to find all possible roots.
Next, use synthetic division to test and find the first rational root.
Finally, use the quadratic formula to find any other real roots.

Example 1:

$P(x) = 1x^3 + 5x^2 + 7x + 3 \to p =$ 3 $q =$ 1 $\frac{p}{q} = \frac{\pm 1, \pm 3}{\pm 1}$

$(-3, -1, 1, \cancel{3})$

$$\begin{array}{r|rrrr} 1 & 1 & 5 & 7 & 3 \\ & & 1 & 6 & 13 \\ \hline & 1 & 6 & 13 & 16 \end{array}$$

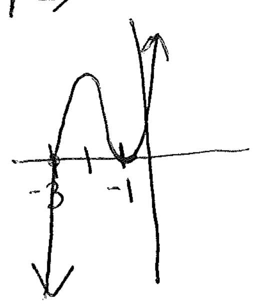
$$\begin{array}{r|rrrr} -1 & 1 & 5 & 7 & 3 \\ & & -1 & -4 & -3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$(1x^2+4x+3)$

$(x+1)(x^2+4x+3)$

$(x+1)\;(x+3)(x+1)$

$(x+1)^2(x+3)$



Example 2:

$9x^3 - 30x^2 + 23x + 2 = 0$ $p =$ ________ $q =$ ________ $\frac{p}{q} =$ ______________________________

Recall from Lesson 4-8 that the complex numbers $a+bi$ and $a-bi$ are conjugates. Similarly, the irrational numbers $a+\sqrt{b}$ and $a-\sqrt{b}$ are conjugates. If a complex number or an irrational number is a root of a polynomial, so is its conjugate.

take note

Theorem Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

When we solve polynomial s, our square roots and complex numbers always come in ______________.

3. A quartic polynomial P(x) has rational coefficients. If $\sqrt{2}$ and $1+i$ are roots of $P(x)=0$.
   What are the two other roots?
   $\downarrow$ $\downarrow$
   $-\sqrt{2}$ $1-i$

4. Write a polynomial function with rational coefficients so the $P(x)=0$ has the given roots.
   $3i$ and $\sqrt{6}$ and the end behavior is fall, fall.

   $-3i$, $-\sqrt{6}$

5. Write a polynomial function with rational coefficients so the $P(x)=0$ has the given roots.
   Cubic with zeros of 4 and $5i$ and the end behavior is fall, rise.

Algebra 2
Worksheet 5.5

Name ____________________
Date ______________ Hour ________

real
imaginary

Find <u>all</u> of the roots of each polynomial function.

1. $P(x) = 2x^3 + 5x^2 + 4x + 1$

q   p

$\frac{\pm 1}{\pm 1, \pm 2}$

$$\begin{array}{r|rrrr} 1 & 2 & 5 & 4 & 1 \\ & & 2 & 7 & 11 \\ \hline & 2 & 7 & 11 & 12 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 2 & 5 & 4 & 1 \\ & & -2 & -3 & -1 \\ \hline & 2 & 3 & 1 & 0 \end{array}$$

$(x+1)(2x^2+3x+1)$

$(x+1)(2x+1)(x+1)$

$(x+1)^2(2x+1)$

$\boxed{x=-1 \quad x=-1/2}$

2. $P(x) = 5x^3 - 11x^2 + 7x - 1$

$\frac{\pm 1}{\pm 1, \pm 5}$

$$\begin{array}{r|rrrr} 1 & 5 & -11 & 7 & -1 \\ & & 5 & -6 & 1 \\ \hline & 5 & -6 & 1 & \emptyset \end{array}$$

$(x-1)(5x^2-6x+1)$

$(x-1)(5x-1)(x-1)$

$(x-1)^2(5x-1)$

$x=1 \quad x=1/5$

3. $P(x) = x^3 - 6x^2 + 13x - 10$

4. $P(x) = x^3 - 5x^2 - x + 5$

5. $P(x) = x^3 - 12x + 16$

6. $P(x) = 4x^3 - 12x^2 - x + 3$

A polynomial function $P(x)$ with rational coefficients has the given roots. Find two additional roots of $P(x) = 0$.

7. $2 + 3i$ and $\sqrt{7}$

$2-3i, -\sqrt{7}$

8. $3-\sqrt{2}$ and $1+\sqrt{3}$

$3+\sqrt{2}, 1-\sqrt{3}$

9. $-4i$ and $6 - i$

$4i, 6+i$

10. $5-\sqrt{6}$ and $-2+\sqrt{10}$

$5+\sqrt{6}, -2-\sqrt{10}$

11. $\sqrt{5}$ and $-\sqrt{13}$

$-\sqrt{5}, \sqrt{13}$

12. $1-\sqrt{10}$ and $2+\sqrt{2}$

$1+\sqrt{10}, 2-\sqrt{2}$

Write a polynomial function $P(x)$ with rational coefficients so that $P(x) = 0$ has the given roots.

13. -5 and $3i$ $-3i$

$(x+5)(x-3i)(x--3i)$

$(x+5)(x-3i)(x+3i)$

14. 5 and $2i$, $-2i$

$(x-5)(x-2i)(x--2i)$

$(x-5)(x-2i)(x+2i)$

15. $\sqrt{3}$, 2, and $-i$

$(x-\sqrt{3})(x$

16. $-\sqrt{7}$ and i

17. You are building a rectangular sandbox for a children's playground. The width of the sandbox is 4 times its height. T e length of the sandbox is 8 ft more than 2 times its height. You have 40 ft^3 of sand available to fill this sandbox. What are the dimensions of the sandbox?