

To solve logarithmic equation, remember that if two logs with the same base are equal, their insides must also be equal.

**Example 1** Solve:  $\log_2(x - 1) = \log_2(4)$ .

$$x - 1 = 4$$

Set the inside of the logs equal to each other.

$$x = 5$$

Add 1 to each side.

The answer is  $x = 5$ .

Sometimes you need to combine logs before solving the equation.

**Example 2** Solve:  $\log_{10}(x + 1) + \log_{10}(x - 1) = \log_{10}(8)$

$$\log_{10}((x + 1)(x - 1)) = \log_{10}8$$

Use the Product Rule for Logarithms to simplify the left-hand side of the equation.

$$(x + 1)(x - 1) = 8$$

Set the inside of the logs equal to each other.

$$x^2 - 1 = 8$$

Simplify.

$$x^2 = 9$$

Add 1 to each side.

$$x = -3, x = 3$$

Take the square root of each side.

Recall that the inside of a logarithm cannot be negative. If  $x$  equals  $-3$ , then  $\log_{10}(x + 1)$  would equal  $\log_{10}(-2)$ , which does not exist. Therefore the only solution is  $x = 3$ .

When the logarithm equals a number, rewrite the logarithm as an exponential equation, then solve.

**Example 3** Solve:  $\log_2(x + 2) = 5$

$$\log_2(x + 2) = 5$$

$$2^5 = x + 2$$

Rewrite the logarithm as an exponential equation.

$$32 = x + 2$$

Simplify.

$$x = 30$$

Subtract 2 from each side.

#### IV. Answer Key

1.  $x = -2$
2.  $x = 53$
3.  $x = 7$
4. no solution
5.  $x = 3.5$
6.  $x = 11/4$
7.  $x = -2$
8.  $x = 28$
9.  $x = 9$
10.  $x = 1$
11.  $x = 2$
12. no solution
13. no solution
14.  $x = 7$
15.  $x = 12.5$
16. no solution
17.  $x = 2$
18.  $x = 2$
19.  $x = -4.5$
20.  $x = 29$
21.  $x = -1/3$
22.  $x = 7.5$
23.  $x = 4$
24.  $x = 17/16$
25.  $x = 4$
26.  $x = 6$
27.  $x = -4$  cannot be a solution to the equation; the inside of a logarithm cannot be negative.

Solve the following logarithmic equations. If there is no solution, so state. SHOW YOUR WORK!!!!

1.  $\log_3(4 - x) = \log_3(x + 8)$

2.  $\log_4(x + 2) = \log_4(55)$

3.  $\log_2(2x + 1) = \log_2(15)$

4.  $\log_5(x + 1) = \log_5(2x + 7)$

5.  $\log_3(x + 2) = \log_3(3x - 5)$

6.  $\log_7(x + 3) = \log_7(5x - 8)$

7.  $\log_5(-x + 1) = \log_5(5 + x)$

8.  $\log_8(2x + 4) = \log_8(60)$

9.  $\log_4(x + 1) = \log_4(10)$

10.  $\log_4(3x + 1) = \log_4(2x)$

11.  $\log_2(x + 2) + \log_2(x + 1) = \log_2(x) + \log_2(x + 4)$

12.  $\log_2(x) + \log_2(x + 1) = \log_2(-4x - 6)$

13.  $\log_2(x - 2) + \log_2(x - 5) = \log_2(x - 1) + \log_2(x + 6)$

**14.**  $\log_2(x) + \log_2(x - 6) = \log_2(2x - 7)$

**20.**  $\log_3(x - 2) = 3$

**15.**  $\log_2(x - 2) + \log_2(x - 8) = \log_2(x + 1) + \log_2(x - 9)$

**16.**  $2 \cdot \log_3(x + 1) = \log_3(x + 2) + \log_3(x - 3)$

**21.**  $\log_2(2 + 3x) = 0$

**17.**  $2 \cdot \log_4(x + 3) = \log_4(25)$

**22.**  $\log_2(2x + 1) = 4$

**18.**  $3 \cdot \log_2(x + 1) = \log_2(27)$

**23.**  $\log_4(17x - 4) = 3$

**19.**  $\log_2(x + 5) = -1$

**24.**  $\log_4(x - 1) = -2$