



Algebra 2

Name: _____

Date: _____ Hour: _____

Notes 5.6

The Fundamental Theorem of Algebra



Theorem The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

The number of **complex** roots of an equation is equal to the _____ of the equation.

The *possible* number of **real** roots of an equation is equal to the possible number of _____ of the related graph.

If the degree of the equation is *even*, the end behaviors of the graph are _____ and the possible number of x-intercepts or real roots is equal to the _____ or all _____ numbers less than the degree.

If the degree of the equation is *odd*, the end behaviors of the graph are _____ and the possible number of x-intercepts or real roots is equal to the _____ or all _____ numbers less than the degree.

For each equation, state the number of complex roots (c.r.), the possible number of real roots (p. # real r.), and list all possible rational roots (p.rat. roots).

HW # 1-4

1. $2x^3 + 2x^5 - x^2 + 12x - 8 = 0$

2. $3x^6 - 8x^4 + 2x^2 - 10 = 0$

c.r. _____

c.r. _____

p.# real r. _____

p.# real r. _____

p. rat. roots _____

p. rat. roots _____

When finding the roots of a polynomial you must read your directions carefully!!!

Are they asking you to find: a) all of the **rational** roots

b) all of the **real** roots

c) all of the **complex** roots

Find all of the **rational** roots of the polynomial.

3. $P(x) = x^4 - 4x^3 + x^2 + 12x - 12$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 1 & 12 & -12 \\ & & 2 & -4 & -6 & 12 \\ \hline & 1 & -2 & -3 & 6 & 0 \end{array}$$

$$x^2(x-2) - 3(x-2) \\ (x-2)(x^2-3)$$

2

$$(x-2)(x^3 - 2x^2 - 3x + 6) \\ (x-2)(x-2)(x^2-3)$$

$$x^2 - 3 = 0 \\ \sqrt{x^2} = \pm\sqrt{3}$$

Find all of the **real** zeros of the polynomial.

4. $P(x) = x^3 - x^2 - 3x - 9$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -3 & -9 \\ & & 3 & 6 & 9 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$$x^2 + 2x + 3 = 0$$

$$x^2 + 2x + 1 = -3 + 1$$

$$\sqrt{(x+1)^2} = \sqrt{-2}$$

$$x+1 = \pm\sqrt{2}i$$

$$x = -1 \pm\sqrt{2}i$$

$$(x-3)(x^2 + 2x + 3)$$

$$x = 3$$

Find all the **complex** roots of the polynomial.

5. $P(x) = 2x^3 + x^2 + 1$

$$\begin{array}{r|rrrr} -1 & 2 & 1 & 0 & 1 \\ & & -2 & +1 & -1 \\ \hline & 2 & -1 & +1 & 0 \end{array}$$

$$-1, \frac{1 \pm \sqrt{7}i}{4}$$

$$(x+1)(2x^2 - 1x + 1)$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$\frac{1 \pm \sqrt{1 - 4(2)(1)}}{4} = \frac{1 \pm \sqrt{7}i}{4}$$

For each equation, state the number of complex roots, the possible number of real roots, and list all the possible rational roots.

1. $2x^4 + x^2 - x + 6 = 0$

2. $4x^5 - 5x^4 + x^3 - 2x^2 + 2x - 6 = 0$

c.r. _____

c.r. _____

p.# real r. _____

p.# real r. _____

p. rat. roots _____

p. rat. roots _____

3. $6 - x + 2x^3 - x^3 + x^4 - 8x^5 = 0$

4. $2x^3 - 6x^2 - 7x - 12 = 0$

c.r. _____

c.r. _____

p.# real r. _____

p.# real r. _____

p. rat. roots _____

p. rat. roots _____

Find all the **rational** roots of each equation.

5. $P(x) = 3x^3 - 10x^2 + 9x - 2$

6. $P(x) = x^3 - 10x - 12$

Find all of the **real** roots of the polynomial.

7. $P(x) = 2x^3 + 5x^2 - x - 4$

8. $P(x) = x^4 - 2x^3 - x^2 - 4x - 6$

Find all of the **complex** zeros of the polynomial.

9. $P(x) = x^4 - 3x^3 - x^2 - 9x - 12$

10. $P(x) = x^5 - 3x^4 - 8x^3 - 8x^2 - 9x - 5$