Function Operations

Content Standards

F.BF.1.b Combine standard function types using arithmetic operations.

F.BF.1.c Compose functions.

Objectives To add, subtract, multiply, and divide functions
To find the composite of two functions

Essential Understanding You can add, subtract, multiply, and divide functions based on how you perform these operations for real numbers. One difference, however, is that you must consider the domain of each function.

take note

Key Concepts Function Operations

Addition	$(f + g)(x) = f(x) + g(x)$
Subtraction	$(f - g)(x) = f(x) - g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of *both* f and g. Also, the domain of the quotient function does not contain any x-value for which $g(x) = 0$.

Problem 1 Adding and Subtracting Functions

Let $f(x) = 4x + 7$ and $g(x) = \sqrt{x} + x$. What are $f + g$ and $f - g$? What are their domains?

$$(f + g)(x) = f(x) + g(x) = (4x + 7) + (\sqrt{x} + x) = 5x + \sqrt{x} + 7$$

$$(f - g)(x) = f(x) - g(x) = (4x + 7) - (\sqrt{x} + x) = 3x - \sqrt{x} + 7$$

Think

What determines the domain of g?
Because there is a square root of x, x must be ≥ 0.

The domain of f is the set of all real numbers. The domain of g is all $x \geq 0$. The domain of both $f + g$ and $f - g$ is the set of numbers common to the domains of both f and g, which is all $x \geq 0$.

Got It? 1. Let $f(x) = 2x^2 + 8$ and $g(x) = x - 3$. What are $f + g$ and $f - g$? What are their domains?

$f+g(x) = 2x^2+x+5$
$(-\infty,\infty)$

$f-g(x) = 2x^2-x+11$
$(-\infty,\infty)$

Problem 2 Multiplying and Dividing Functions

Let $f(x) = x^2 - 9$ and $g(x) = x + 3$. What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

$$(f \cdot g)(x) = f(x) \cdot g(x) = (x^2 - 9)(x + 3)$$
$$= x^3 + 3x^2 - 9x - 27$$

Think

Is the domain of $\frac{f}{g}$ the domain of $x - 3$?
No; The fraction can only be simplified and the function is only defined when $g(x) \neq -3$.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 9}{x + 3} = \frac{(x - 3)(x + 3)}{x + 3} = x - 3, x \neq -3$$

The domain of both f and g is the set of real numbers, so the domain of $f \cdot g$ is also the set of real numbers.
The domain of $\frac{f}{g}$ is the set of all real numbers except $x \neq -3$, because $g(-3) = 0$. The definition of $\frac{f}{g}$ requires that you consider the zero denominator in the *original* expression for $\frac{f(x)}{g(x)}$ despite the fact that the simplified form has the domain all real numbers.

Got It? 2. Let $f(x) = 3x^2 - 11x - 4$ and $g(x) = 3x + 1$. What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

$f \cdot g(x) = (3x^2 - 11x - 4)(3x + 1)$
arn $(-\infty, \infty)$

$\frac{3x^2 - 11x - 4}{3x + 1} = \frac{(3x+1)(x-4)}{(3x+1)}$

$(-\infty, -1/3) \cup (-1/3, \infty)$

Take note — Key Concept Composition of Functions

The composition of function g with function f is written as $g \circ f$ and is defined as $(g \circ f)(x) = g(f(x))$. The domain of $g \circ f$ consists of the x-values in the domain of f for which $f(x)$ is in the domain of g.

$(g \circ f)(x) = g(f(x))$
1. Evaluate $f(x)$ first.
2. Then use $f(x)$ as the input for g.

Function composition is not commutative since $f(g(x))$ does not always equal $g(f(x))$.

Problem 3 Composing Functions

GRIDDED RESPONSE

Let $f(x) = x - 5$ and $g(x) = x^2$. What is $(g \circ f)(-3)$?

Think

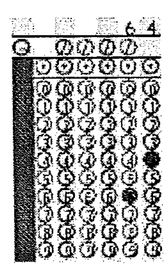
Which function is substituted into the other?
Use $f(x)$ as the input for g.

Method 1

$$(g \circ f)(x) = g(f(x))$$
$$= g(x - 5) = (x - 5)^2$$
$$(g \circ f)(-3) = (-3 - 5)^2$$
$$= (-8)^2$$
$$= 64$$

Method 2

$$(g \circ f)(-3) = g(f(-3))$$
$$= g(-3 - 5)$$
$$= g(-8)$$
$$= (-8)^2$$
$$= 64$$



Got It? 3. What is $(f \circ g)(-3)$ for the functions f and g defined in Problem 3?

Example 5 Writing a Function as a Composite

Let $h(x) = \sqrt{3x^2 + 1}$. Write h as a composition of functions in two different ways.

$f(x) = \sqrt{x}$

$g(x) = 3x^2 + 1$

$h(x) = f(g(x))$

In Exercises 33–38, write the given function as the composite of two functions, neither of which is the identity function, $f(x) = x$. (There may be more than one possible answer.)

33. $f(x) = \sqrt[3]{x^2 + 2}$

$g(x) = \sqrt[3]{x}$

$h(x) = x^2 + 2$

$f(x) = g(h(x))$

35. $h(x) = (7x^3 - 10x + 17)^7$

$f(x) = x^7$

$g(x) = 7x^3 - 10x + 17$

$h(x) = f(g(x))$

37. $f(x) = \dfrac{1}{3x^2 + 5x - 7}$

$g(x) = \frac{1}{x}$

$h(x) = 3x^2 + 5x - 7$

$f(x) = g(h(x))$

For Exercises 41–44, complete the given tables by using the values of the functions f and g given below.

x	$f(x)$
1	3
2	5
3	1
4	2
5	3

x	$g(x)$
1	5
2	4
3	4
4	3
5	2

$g \circ f(x)$

41.

x	$(g \circ f)(x)$
1	4
2	2
3	5
4	4
5	4

42.

x	$(f \circ g)(x)$
1	3
2	2
3	2
4	1
5	5

43.

x	$(f \circ f)(x)$
1	1
2	3
3	3
4	5
5	1

44.

x	$(g \circ g)(x)$
1	2
2	3
3	3
4	4
5	4

In Exercises 21–24, find the rule of the function $g \circ f$ and its domain and the rule of $f \circ g$ and its domain.

21. $f(x) = x^2$ $\quad g(x) = x + 3$

$g(f(x)) = g \circ f(x) = x^2 + 3$

$f(g(x)) = (x+3)^2$

$x^2 + 6x + 9$

23. $f(x) = \frac{1}{x}$ $\quad g(x) = \sqrt{x}$

$f(g(x)) = \dfrac{1}{\sqrt{x}}$ $\quad (0, \infty)$

$g(f(x)) = \sqrt{\dfrac{1}{x}}$ $\quad (0, \infty)$

Let $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$. Perform each function operation and then find the domain.

21. $(f + g)(x)$

22. $(f - g)(x)$

23. $(g - f)(x)$

24. $(f \cdot g)(x)$

25. $\frac{f}{g}(x)$

26. $\frac{g}{f}(x)$

Let $f(x) = 2x + 5$ and $g(x) = x^2 - 3x + 2$. Perform each function operation and then find the domain.

47. $f(x) + g(x)$

48. $3f(x) - 2$

49. $g(x) - f(x)$

50. $-2g(x) + f(x)$

51. $f(x) - g(x) + 10$

52. $4f(x) + 2g(x)$

53. $-f(x) + 4g(x)$

54. $f(x) - 2g(x)$

55. $f(x) \cdot g(x)$

56. $-3f(x) \cdot g(x)$

57. $\frac{f(x)}{g(x)}$

58. $\frac{5f(x)}{g(x)}$