Key

6-1 Roots and Radical Expressions

Content Standard
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Objective To find *n*th roots

Take Note

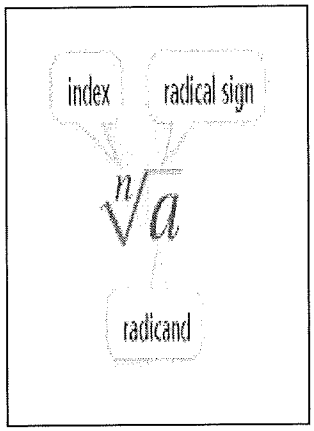
Key Concept The *n*th Root

If $a^n = b$, with a and b real numbers and n a positive integer, then a is an **nth root** of b.

If *n* is odd...
there is one real nth root of b, denoted in radical form as $\sqrt[n]{b}$.

If *n* is even...
- and b is positive, there are two real nth roots of b. The positive root is the **principal root** (or principal nth root) and its symbol is $\sqrt[n]{b}$. The negative root is its opposite, or $-\sqrt[n]{b}$.
- and b is negative, there are no real nth roots of b.

The only nth root of 0 is 0.



You use a radical sign to indicate a root. The number under the radical sign is the **radicand**. The **index** gives the degree of the root.

$\sqrt[4]{}$

Problem 1 Finding All Real Roots

A **What are the real cube roots of 0.008, −1000, and $\frac{1}{27}$?**

$(.2)^3 = .2$ | $(-10)^3 = -10$ | $\left(\frac{1}{3}\right)^3 = \frac{1}{3}$

B **What are the real fourth roots of 1, −0.0001, and $\frac{16}{81}$?**

1^4 $(-1)^4$ so 1 and −1 | no real roots every neg | $\left(\frac{2}{3}\right)^4 = \frac{2}{3}$ $\left(-\frac{2}{3}\right)^4 = -\frac{2}{3}$

Got It? 1. a. What are the real fifth roots of 0, −1, and 32?
b. What are the real square roots of 0.01, −1, and $\frac{36}{121}$?
c. **Reasoning** Explain why a negative real number b has no real nth roots if n is even.

a) 0, −1, 2

b) ±.1, no real root, $\pm\frac{6}{11}$

Any neg number x by itself an even # of times will always be +
Therefore there can be no real nth root when n is even for a negative number b

According to the Fundamental Theorem of Algebra, $x^4 - 1 = 0$ has four roots, only two of which are real. In this chapter, the focus is on real roots only.

Problem 2 Finding Roots

What is each real-number root?

A $\sqrt[3]{-8}$

-2

B $\sqrt{0.04}$

$.2$

C $\sqrt[4]{-1}$

$\varnothing$

D $\sqrt{(-2)^2}$

$\sqrt{4}=2$

Got It? 2. What is each real-number root?

a. $\sqrt[3]{-27}$

-3

b. $\sqrt[4]{-81}$

$\varnothing$

c. $\sqrt{(-7)^2}$

7

d. $\sqrt{-49}$

$\varnothing$

It is tempting to conclude that $\sqrt[n]{a^n} = a$ for all real numbers a, but part (d) of Problem 2 shows that this is not the case. If n is even, then $\sqrt[n]{a^n}$ is positive even if a itself is negative.

Take Note

Property ***n*th Roots of *n*th Powers**

For any real number a, $\sqrt[n]{a^n} = \begin{cases} a \text{ if } n \text{ is odd} \\ |a| \text{ if } n \text{ is even} \end{cases}$

It is easy to overlook this rule for simplifying radicals. It is particularly important that you remember it when the radicand contains a variable expression. You must *include* the absolute value when n is even, and you must *omit* it when n is odd.

Problem 3 Simplifying Radical Expressions

What is a simpler form of each radical expression?

A $\sqrt{16x^8}$

$|4x^4| = 4x^4$

B $\sqrt[3]{a^6b^9}$

a^2b^3

C $\sqrt[4]{x^8y^{12}}$

$x^2|y^3|$

Got It? 3. What is the simplified form of each radical expression?

a. $\sqrt{81x^4}$

$9x^2$

b. $\sqrt[3]{a^{12}b^{15}}$

a^4b^5

c. $\sqrt[4]{x^{12}y^{16}}$

$|x^3|y^4$

Problem 4 Using a Radical Expression

Academics Some teachers adjust test scores when a test is difficult. One teacher's formula for adjusting scores is $A = 10\sqrt{R}$, where A is the adjusted score and R is the raw score. If the raw scores on one test range from 36 to 90, what is the range of the adjusted scores?

Think	Write
You have to adjust the lowest raw score and the highest raw score.	$10\sqrt{36} = 10(6) = 60$ $10\sqrt{90} \approx 10(9.487) = 94.87 \approx 95$
The other adjusted scores must be between the lowest and highest adjusted scores.	The adjusted scores range from 60 to 95.

Got It? 4. In Problem 4, what are the adjusted scores for raw scores of 0 and 100?

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice Find all the real square roots of each number. See Problem 1.

10. 225
11. 0.0049
12. $-\frac{1}{121}$
13. $\frac{64}{169}$

Find all the real cube roots of each number.

14. -64
15. 0.125
16. $-\frac{27}{216}$
17. 0.000343

Find all the real fourth roots of each number.

18. 16
19. -16
20. 0.0081
21. $\frac{10{,}000}{81}$

Find each real root. See Problem 2.

22. $\sqrt{36}$ 23. $\sqrt{0.25}$ 24. $-\sqrt[3]{64}$ 25. $\sqrt[3]{-27}$

Simplify each radical expression. Use absolute value symbols when needed. See Problem 3.

26. $\sqrt{16x^2}$ 27. $\sqrt[3]{27y^6}$ 28. $\sqrt[4]{x^{20}y^{28}}$ 29. $\sqrt[5]{32y^{10}}$

30. **Grades** In many classes, a passing test grade is 70. Using the formula $A = 10\sqrt{R}$, what raw score would a student need to get a passing grade after her score is adjusted? See Problem 4.

B Apply Find the two real solutions of each equation.

31. $x^2 = 100$ 32. $x^4 = 1$ 33. $x^2 = 0.25$ 34. $x^4 = \frac{16}{81}$