3.6 Inverse Relations and Functions

Use the relation {(4, 5), (6, 7), (12, 20), (8, 3), (2, 7)}. Write the domain and range of the relation.

Domain 4, 6, 12, 8, 2
{2, 4, 6, 8, 12}

Range
{3, 5, 7, 20}

inverse (noun) in VURS

Related Words: opposite, reverse

Math Usage: The inverse of a function is found by reversing the order of the elements in the ordered pairs.

Example: The inverse of the function {(1, 2), (2, 4), (3, 6)} is {(2, 1), (4, 2), (6, 3)}.

3. Complete the diagram below. Use relation r {(0, 1), (2, 3), (4, 1), (8, 3)}.

Relation r

Domain	Range
0, 2, 4, 8	1, 3

Inverse of Relation r

Domain	Range
1, 3	0, 2, 4, 8

Problem 1 Finding the Inverse of a Relation

Got It? What are the graphs of t and its inverse?

4. Complete the table of values for the inverse of relation t.

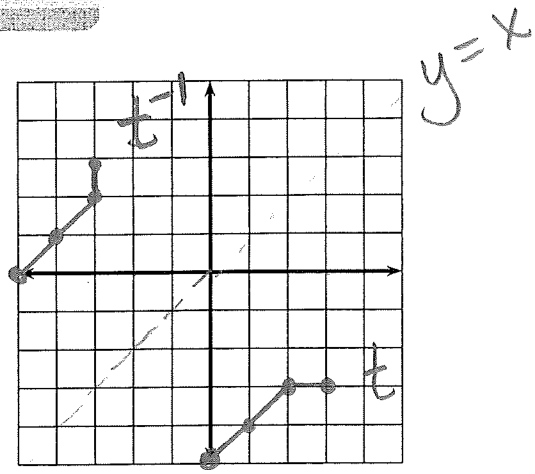
Relation t

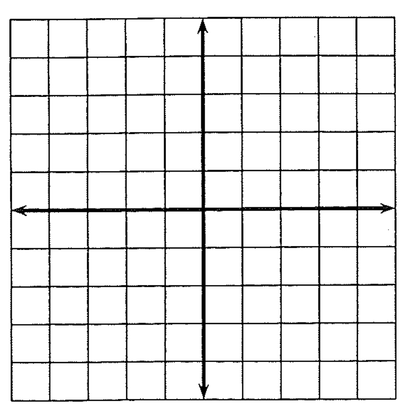
x	y
0	−5
1	−4
2	−3
3	−3

→

Inverse of Relation t

x	y
−5	0
−4	1
−3	2
−3	3





$X \to Y$

Problem 2 Finding an Equation for the Inverse

Got It? What is the inverse of $y = 2x + 8$?

6. Switch the x and y values in the function.

function $y = 2x + 8$

inverse $x = 2y + 8$

$x = 2y + 8$

$y^{-1} = \frac{1}{2}x - 4$

7. Solve the inverse equation for y.

$y = \frac{1}{2}x - 4$

$f^{-1}(x) = \frac{1}{2}x - 4$

$\frac{x-8}{2} = \frac{2y}{2}$

$\frac{1}{2}x - 4 = y$

Problem 3 Graphing a Relation and Its Inverse

Got It? What are the graphs of $y = 2x + 8$ and its inverse?

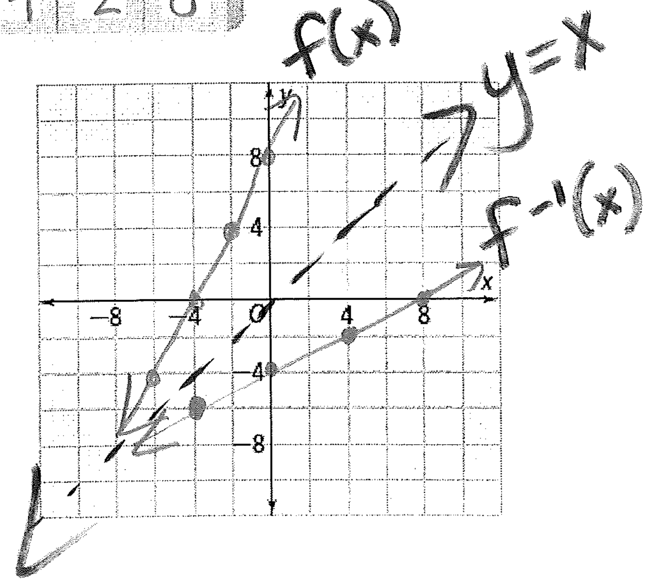
8. Complete the table for $y = 2x + 8$.

x	−6	−4	−2	0
y	−4	0	4	8

9. Complete the table for the inverse of $y = 2x + 8$.

x	−4	0	4	8
y	−6	−4	−2	0

10. Plot and draw a line through the points from the $y = 2x + 8$ table.
11. On the same grid, plot and draw a line through the points from the inverse of $y = 2x + 8$ table.
12. Draw a dashed line to show the line that reflects the equation $y = 2x + 8$ to its inverse.



Graphs of Inverse Relations

Let f be a function. If (a, b) is a point on the graph of f, then (b, a) is a point on the graph of its inverse.

The graph of the inverse of f is a reflection of the graph of f across the line

$y = x$

take note

Key Concept Composition of Inverse Functions

If f and f^{-1} are inverse functions, then $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domains of f and f^{-1}, respectively.

Problem 6 Composing Inverse Functions

Got It? Let $g(x) = \frac{4}{x+2}$. What is $g^{-1}(x)$?

$\boxed{g^{-1}(x) = \frac{4}{x} - 2}$

$y = \frac{4}{x+2}$

$\frac{x}{1} = \frac{4}{y+2}$

$\frac{4}{x} = \frac{x(y+2)}{x}$

$\frac{4}{x} = y+2$

$\frac{4}{x} - 2 = y$

$\frac{4}{x} - 2 = y^{-1}$

Example 3 Finding an Inverse from an Equation

Find $g(x)$, the inverse of $f(x) = 3x - 2$.

$y = 3x-2$

$x = 3y-2$

$\frac{x+2}{3} = \frac{3y}{3}$

$\frac{x}{3} + \frac{2}{3} = y^{-1}$

Example 4 Finding an Inverse from an Equation

Find the inverse of $f(x) = x^2 + 4x$.

$y = x^2+4x$

$x = y^2+4y$

Determining Whether an Inverse is a Function

The inverse of a function is also a function if every input of the inverse corresponds to exactly one output. This means that in the original function, every output corresponds to exactly one input. A function that has this property is called a **one-to-one** function.

One-to-One Functions

A function f is one-to-one if

$$f(a) = f(b)$$

implies that $a = b$.

If a function is one-to-one, then its inverse is also a function.

NOTE By the definition of a function, $a = b$ implies that $f(a) = f(b)$.

Determining Whether a Graph is One-to-One

In Example 1, the points $(2, 4)$ and $(-2, 4)$ are both on the graph of f. These two points have different inputs and the same output, so f is not one-to-one. Also, the points $(2, 4)$ and $(-2, 4)$ lie on the same horizontal line, which suggests a graphical test for whether a function is one-to-one.

Horizontal Line Test

A function f is one-to-one if and only if no horizontal line intersects the graph of f more than once.

Example 5 Using the Horizontal Line Test

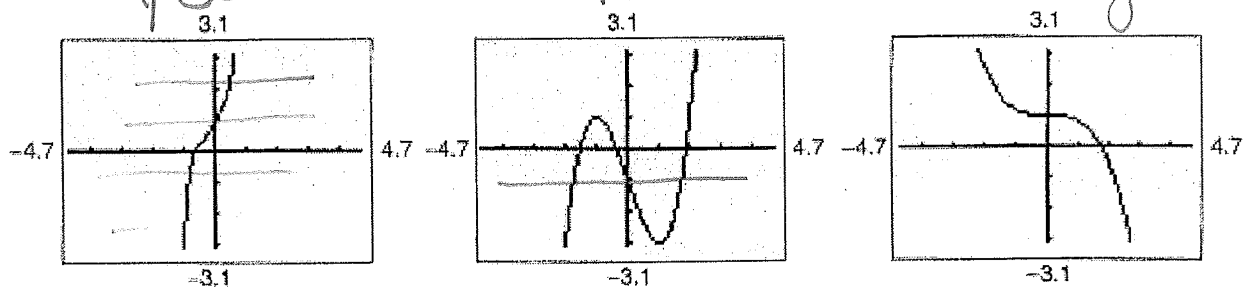
Graph each function below and determine whether the function is one-to-one. If so, graph its inverse function.

a. $f(x) = 7x^5 + 3x^4 - 2x^3 + 2x + 1$
b. $g(x) = x^3 - 3x - 1$
c. $h(x) = 1 - 0.2x^3$

Solution

Complete graphs of each function are shown below.

yes NO yes



a. b. c.

Restricting the Domain

For a function that is not one-to-one, it is possible to produce an inverse function by considering only a part of the function that is one-to-one. This is called **restricting the domain.**

Example 6 Restricting the Domain

Find an interval on which the function $f(x) = x^2$ is one-to-one, and find f^{-1} on that interval.

Example 7 Verifying the Inverse of a Function

Let

$$y = \frac{5}{2x-4}$$

$$f(x) = \frac{5}{2x-4} \quad \text{and} \quad g(x) = \frac{4x+5}{2x}$$

Use composition to verify that f and g are inverses of each other.

$$f(g(x)) = \frac{5}{2\left(\frac{4x+5}{2x}\right) - 4}$$

$$= x$$

$$g(f(x))$$

$$\frac{4\left(\frac{5}{2x-4}\right)+5}{2(2x-4)}$$

$$= x$$

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6-7 Practice

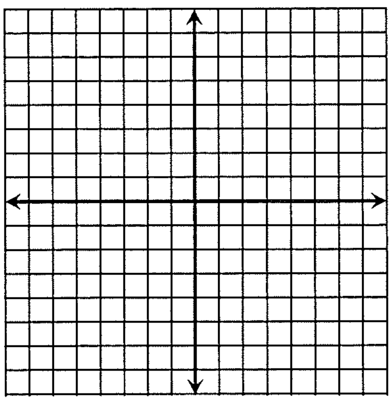
Form G

Inverse Relations and Functions

Find the inverse of each relation. Graph the given relation and its inverse.

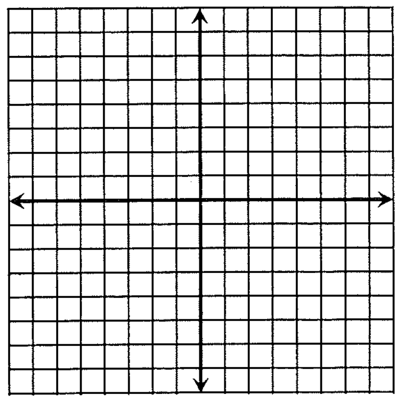
1.

x	−2	−1	0	1
y	−3	−2	−1	0



2.

x	0	1	2	3
y	−3	−1	0	−2



Find the inverse of each function. Is the inverse a function?

5. $y = x^2 + 2$

6. $y = x + 2$

7. $y = 3(x + 1)$

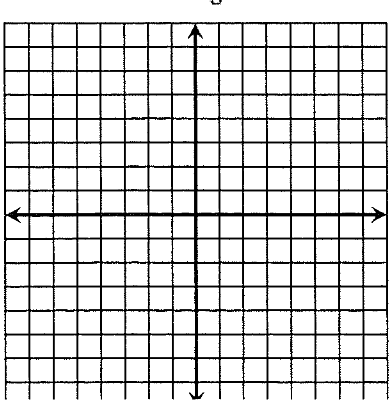
14. $y = 3x^2 - 2$

15. $y = (x + 4)^2 - 4$

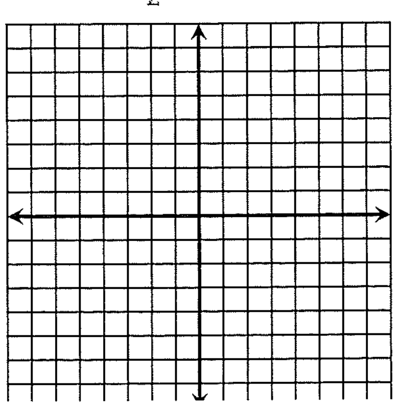
16. $y = -x^2 + 4$

Graph each relation and its inverse.

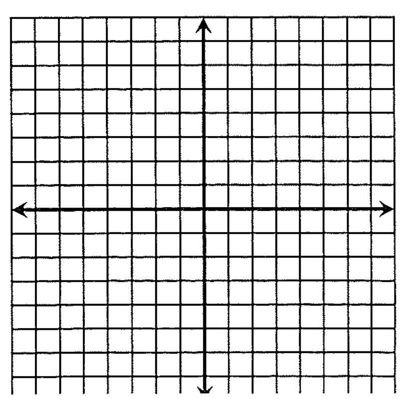
17. $y = \frac{x + 3}{3}$



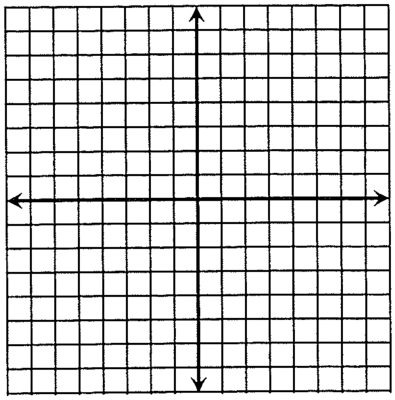
18. $y = \frac{1}{2}x + 5$



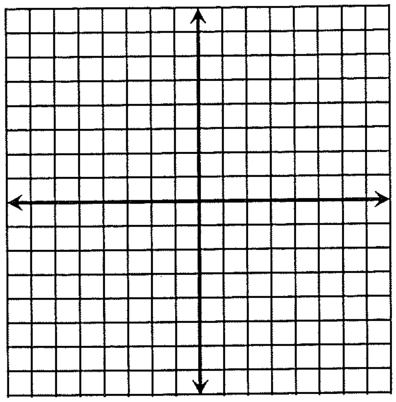
19. $y = 2x + 5$



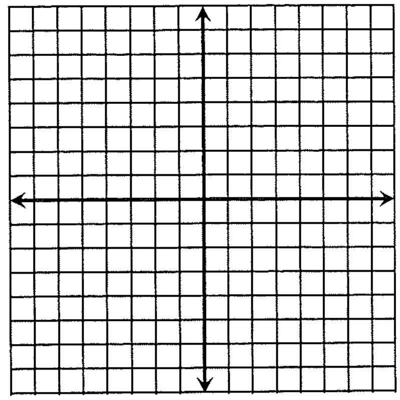
20. $y = \frac{1}{2}x^2$



21. $y = (x + 2)^2$



22. $y = (2x - 1)^2 - 2$



For each function, find the inverse and the domain and range of the function and its inverse. Determine whether the inverse is a function.

23. $f(x) = \frac{1}{6}x$

24. $f(x) = -\frac{1}{5}x + 2$

25. $f(x) = x^2 - 2$

26. $f(x) = x^2 + 4$

27. $f(x) = \sqrt{x - 1}$

28. $f(x) = \sqrt{3x}$

In Exercises 45–50, use composition to show that *f* and *g* are inverses of each other (see Example 7).

45. $f(x) = x + 1 \quad g(x) = x - 1$

46. $f(x) = 2x - 6 \quad g(x) = \frac{x}{2} + 3$

47. $f(x) = \frac{1}{x + 1} \quad g(x) = \frac{1 - x}{x}$