Algebra 2 Name: ______________________

Section 5.2 – Notes and Examples Date: ______________ Hour: ______

Polynomials, Linear Factors, and Zeros

You should be able to write a polynomial in complete, factored form.

Example 1 – Write each polynomial in factored form.

a. $x^3 + x^2 - 12x = 0$

$x(x^2 + x - 12) = 0$

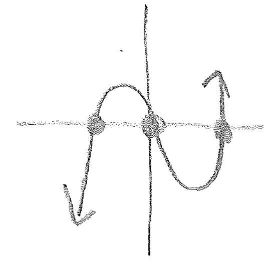
$x(x+4)(x-3) = 0$

b. $7x^3 - 7x$

$7x(x^2 - 1)$

$7x(x-1)(x+1)$

$0 \quad +1 \quad -1$



The factors of a polynomial are important for several reasons, and each factor has multiple meanings.

take note

Key Concepts Roots, Zeros, and x-intercepts

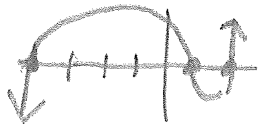
The following are equivalent statements about a real number b and a polynomial

- $x - b$ is a linear factor of the polynomial $P(x)$.
- b is a zero of the polynomial function $y = P(x)$.
- b is a root (or solution) of the polynomial equation $P(x) = 0$.
- b is an x-intercept of the graph of $y = P(x)$.

Example 2 – Find the zeros of each polynomial function. Remember, a "zero" means that the y-value is equal to zero!

a. $y = (x+4)(x-1)(x-2)$ x^3 L.C +

$x = -4 \quad x = 1 \quad x = 2$

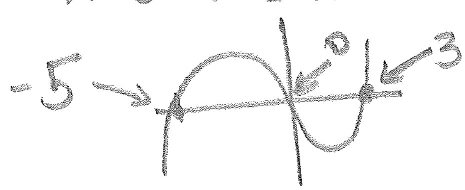


b. $y = x(x-3)(x+5)$ x^3 L.C +

$x = 0 \quad x = 3 \quad x = -5$

$(x^2 - 3)(x+5)$

x^3



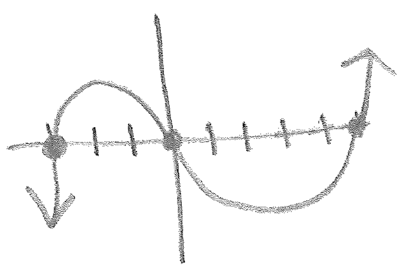
Combine the steps in Examples 1 and 2!

Example 3 – Find the zeros of the polynomial function $y = x^3 - 2x^2 - 15x$.

$= x(x^2 - 2x - 15)$

$= x(x-5)(x+3)$

$x = 0 \quad x = 5 \quad x = -3$



take note

Theorem Factor Theorem

$(x - a)$ $\quad a$

$(x + 2)$ $\quad -2$

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

If 2 is a zero of the function, then $(x-2)$ is a factor of the function.

If –3 is a zero of the function, then $(x+3)$ is a factor of the function.

Example 4 – Write a polynomial that satisfies the given conditions. Be sure to start with "$y =$ "

a. Cubic; zeros at –2, 2, and 3

$f(x) = (x+2)(x-2)(x-3)$

$y = (x^2-4)(x-3)$

$y = x^3 - 3x^2 - 4x + 12$

b. Quartic; zeros at –3, –2, –1, and 2

$f(x) = (x+3)(x+2)(x+1)(x-2)$

$(x^2-4)(x^2+4x+3)$

$x^4 + 4x^3 + 3x^2 - 4x^2 - 16x - 12$

$y = x^4 + 4x^3 - 1x^2 - 16x - 12$

GRAPHING POLYNOMIAL FUNCTIONS

Once you find the zeros of a polynomial, you can make a graph. To create a complete and fairly accurate graph, you also will need to plot points in between the zeros. Follow these steps:

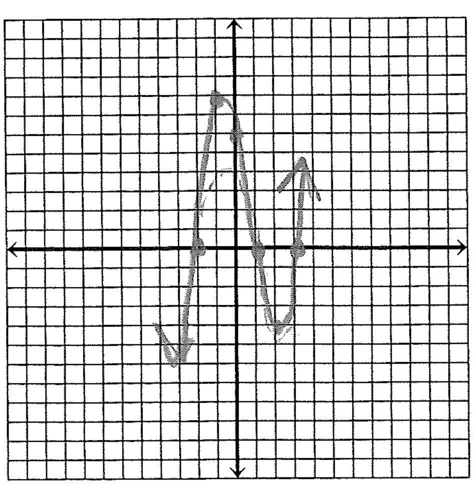
1. Factor the polynomial, if needed.
2. Find the zeros from the linear factors. Plot the zeros as x-intercepts.
3. List the x-intercepts in order from smallest to largest in a t-table, leaving empty rows above and below each one.
4. Pick x-values on either side of each x-intercept, and find the corresponding y-values. Plot those new points.
5. Connect the points with a smooth, polynomial curve. Think about end-behavior!

Example 5 – Graph each polynomial function.

a. $y = (x-1)(x-3)(x+2)$ $+x^3$

$x=1$ $x=3$ $x=-2$

x	plug in x	y
-1	$-2 \cdot -4 \cdot 1$	8
0	$-1 \cdot -3 \cdot 2$	6
2	$1 \cdot -1 \cdot 4$	-4



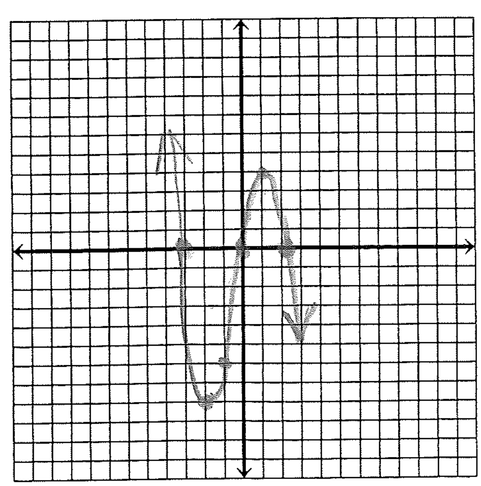
b. $y = -x^3 - x^2 + 6x$

$-x(x^2 + x - 6)$

$-x(x+3)(x-2)$

$0, -3, 2$

x	plug in x	y
-2	$2 \cdot 1 \cdot -4$	-8
-1	$1 \cdot 2 \cdot -3$	-6
1	$-1 \cdot 4 \cdot -1$	4



Example 6 – Find the zeros of each polynomial function. Remember, a "zero" means that the y-value is equal to zero!

a. $y=(x-4)(2x+1)(x+2)^3$

$x=4$ $x=-\frac{1}{2}$ $x=-2$

↓ ↓ ↓

odd odd odd

b. $y=x(x+5)(4x-3)$

↓ ↓ ↓

$x=0$ $x=-5$ $x=3/4$

When one of the linear factors appears more than once in the polynomial (like in Example 6), we say that the ***multiplicity*** of that factor has changed. How does multiplicity affect a graph? It depends on whether the multiplicity is **even** or **odd**:

- If the multiplicity is **even** then the graph will [TOUCH / CROSS] the x-axis.
- If the multiplicity is **odd**, then the graph will [TOUCH / CROSS] the x-axis.

Sometimes instead of "touching" the x-axis, it is described as "bouncing" off the x-axis. The graph will touch the x-axis, and then immediately change directions (a turning point).

Example 8 – State the multiplicity of each factor (you may need to factor first, but then just name the exponent!), and determine whether the graph would touch or cross the x-axis at each zero.
Sketch the graph.

a. $y=x^2(x-4)^3$

↓ ↓

$x=0$ $x=4$

even odd

touch & bounch cross thru

b. $y=x^4-2x^3-8x^2$

$x^2(x^2-2x-8)$

$x^2(x-4)(x+2)$

$x=0$ $x=4$ $x=-2$

even odd odd

+16 cross cross

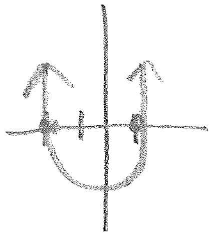
Write each polynomial in factored form. Check by multiplication. See Problem 1.

7. x^3+7x^2+10x **8.** x^3-7x^2-18x **9.** x^3-4x^2-21x

10. x^3-36x **11.** x^3+8x^2+16x **12.** $9x^3+6x^2-3x$

Find the zeros of each function. Then graph the function. See Problem 2.

13. $y = (x-1)(x+2)$ **14.** $y = (x-2)(x+9)$ **15.** $y = x(x+5)(x-8)$



16. $y = (x+1)(x-2)(x-3)$ **17.** $y = (x+1)(x-1)(x-2)$ **18.** $y = x(x+2)(x+3)$

Write a polynomial function in standard form with the given zeros. See Problem 3.

19. $x = 5, 6, 7$ **20.** $x = -2, 0, 1$ **21.** $x = -5, -5, 1$ **22.** $x = 3, 3, 3$

$f(x) = (x-5)(x-6)(x-7)$

Write a polynomial function in standard form with the given zeros. See Problem 3.

19. $x = 5, 6, 7$ **20.** $x = -2, 0, 1$ **21.** $x = -5, -5, 1$ **22.** $x = 3, 3, 3$

Find the zeros of each function. State the multiplicity of multiple zeros.

27. $y = (x+3)^3$

$x = -3$

odd, cross

28. $y = x(x-1)^3$

29. $y = 2x^3 + x^2 - x$

30. $y = 3x^3 - 3x$

31. $y = (x-4)^2$

32. $y = (x-2)^2(x-1)$

33. $y = (2x+3)(x-1)^2$

34. $y = (x+1)^2(x-1)(x-2)$