Name:_____

Date:_____

Hour:_____

3.3 Quadratic Functions

Parabolas

The rule of a **quadratic function** is a polynomial of degree 2. The shape of the graph of a quadratic function is a **parabola.** Three parabolas are shown in Figure 3.3-1, with important points labeled.

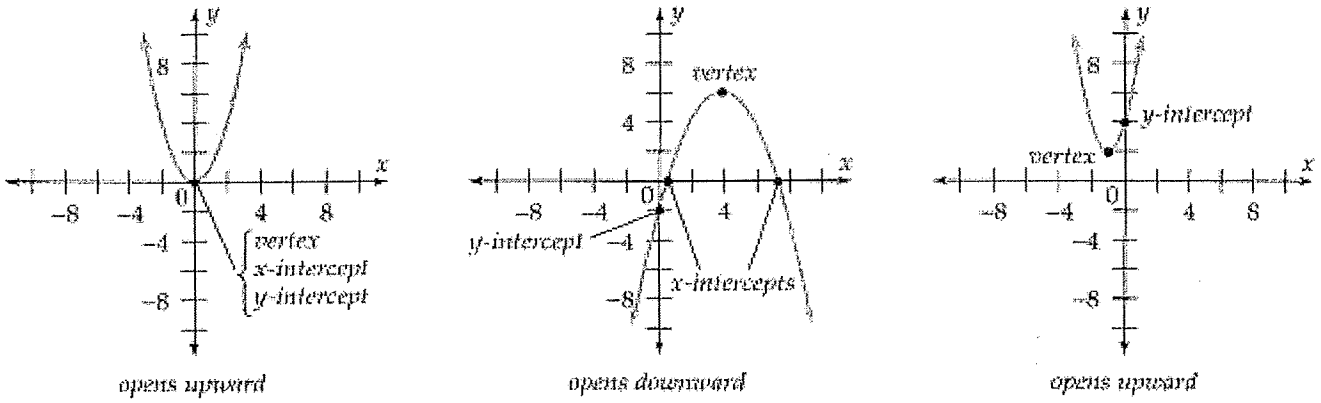


Figure 3.3-1

Notice that the graph of a quadratic function

- can open either upward or downward
- always has a vertex which is either the maximum or minimum
- always has exactly 1 y-intercept
- can have 0, 1, or 2 x-intercepts

A parabola is symmetric about a line through the vertex called the **axis of symmetry.**

Three Forms of a Quadratic Function

A quadratic function can be written in any of the following forms:

Vertex form

Transformation form: $f(x) = a(x - h)^2 + k$

standard form

Polynomial form: $f(x) = ax^2 + bx + c$

x-Intercept form: $f(x) = a(x - s)(x - t)$

where a, b, c, h, k, s, and t are real numbers and $a \neq 0$. If a is positive, the graph opens upward; and if a is negative, the graph opens downward.

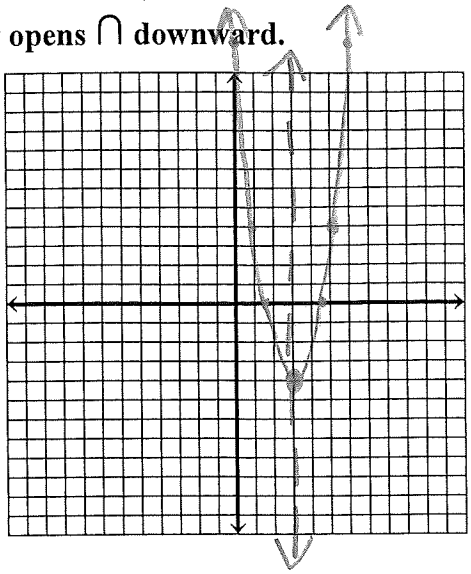
A.) Transformation Form (TF) also called Vertex Form: $f(x) = a(x-h)^2 + k$

↓exact

Example: $f(x) = -6(x-3)^2 + 2$ vertex = (opposite, exact)

↑opposite

What is so nice about this form is the vertex is (3, 2), the axis of symmetry is $x = 3$. Also, the number in front of the parenthesis tells you if it opens upward of downward. This one is -6, so it opens ∩ downward.



Example 1 Transformation Form

For the function $f(x) = 2(x-3)^2 - 4$, find the vertex and the x- and y-intercepts. Then sketch the graph.

Up/~~Down~~: ↑↓ Axis of Symmetry: $x=3$

~~Max~~ or Min: _____ Vertex: $(3,-4)$

Domain: all reals Range: $y \geq -4$

X intercepts: $3 \pm \sqrt{2}$ Y intercept: $(0, 14)$
(set y=0) (set x=0)

$2(x-3)^2 - 4 = 0$
$(x-3)^2 = 2$
$x - 3 = \pm\sqrt{2}$
$x = 3 \pm \sqrt{2}$

$3+\sqrt{2} \approx 4.4$
$3-\sqrt{2} \approx 1.6$

x	y
1	4

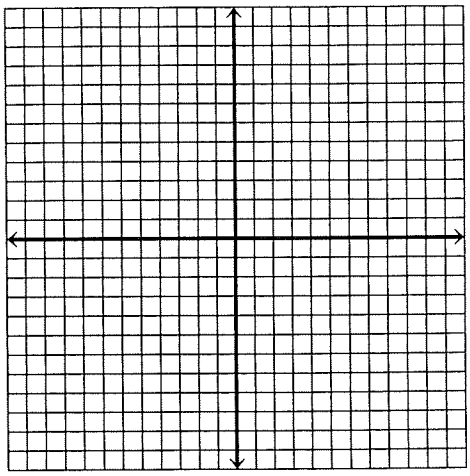
1. $f(x) = 3(x-5)^2 + 2$

Up/Down: _____ Axis of Symmetry: _____

Max or Min: _____ Vertex: _____

Domain: _____ Range: _____

X intercepts: _____ Y intercept: _____
(set y=0) (set x=0)



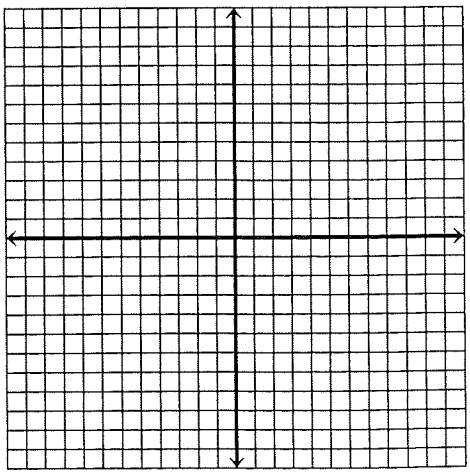
2. $f(x) = -(x-1)^2 + 2$

Up/Down: _____ Axis of Symmetry: _____

Max or Min: _____ Vertex: _____

Domain: _____ Range: _____

X intercepts: _____ Y intercept: _____
(set y=0) (set x=0)



B.) Polynomial Form (PF): $f(x) = ax^2 + bx + c$

Example: $f(x) = 3x^2 + 2x + 1$

↑up ↑ (0,1)

The axis of symmetry = $\frac{-b}{2a}$

This one has the y-intercept of (0,1). Since $a = +3$, it opens ∪ upward.

Example 2 **Polynomial Form**

For the function $f(x) = x^2 + 2x + 3$, find the vertex and the x- and y-intercepts. Then sketch the graph. $x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

x	y
1	1+2+3
2	4+4+3

Up/Down: ∪ Axis of Symmetry: $x = -1$

$(-1)^2 + 2(-1) + 3 = 1 - 2 + 3$

Max or Min: Min Vertex: (-1, 2)

Domain: all Range: $y \geq 2$

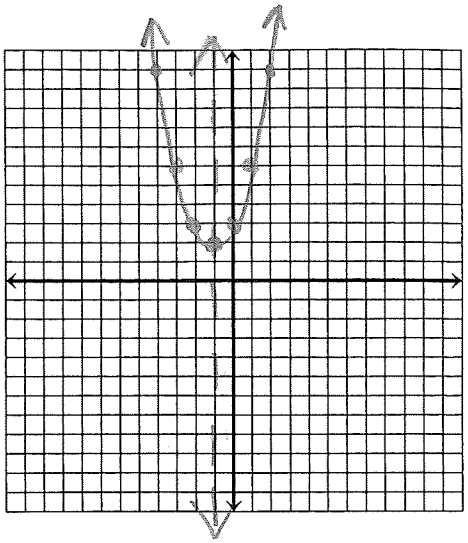
X intercepts: NONE Y intercept: (0, 3)

(set y=0) (set x=0)

$x^2 + 2x + 3 = 0$

$x^2 + 2x + 1 = -3 + 1$

$\sqrt{(x+1)^2} = \sqrt{-2}$



3. $f(x) = x^2 - 6x + 3$

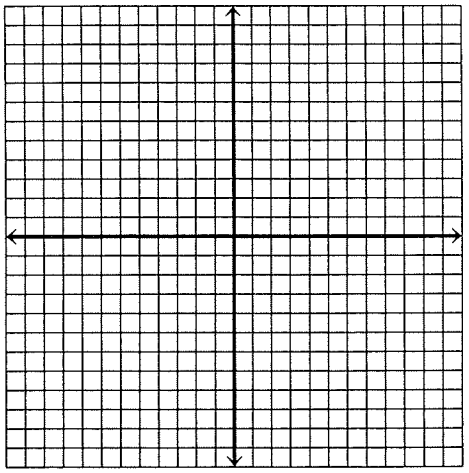
Up/Down: _____ Axis of Symmetry: _____

Max or Min: _____ Vertex: _____

Domain: _____ Range: _____

X intercepts: _____ Y intercept: _____

(set y=0) (set x=0)



4. $f(x) = x^2 + 8x - 1$

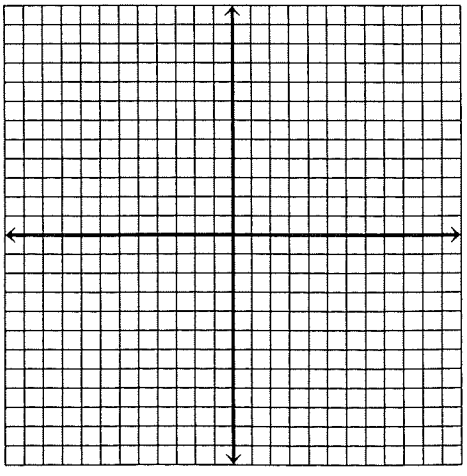
Up/Down: _____ Axis of Symmetry: _____

Max or Min: _____ Vertex: _____

Domain: _____ Range: _____

X intercepts: _____ Y intercept: _____

(set y=0) (set x=0)



C.) X intercept Form (IF): $f(x) = a(x-s)(x-t)$

Example: $f(x) = -2(x-3)(x+5)$

down (3, 0) and (-5, 0) are the x intercepts

Example 3 *x*-Intercept Form

For the graph of the function $f(x) = -\frac{1}{2}(x-4)(x+2)$, find the vertex and the x- and y-intercepts. Then sketch the graph.

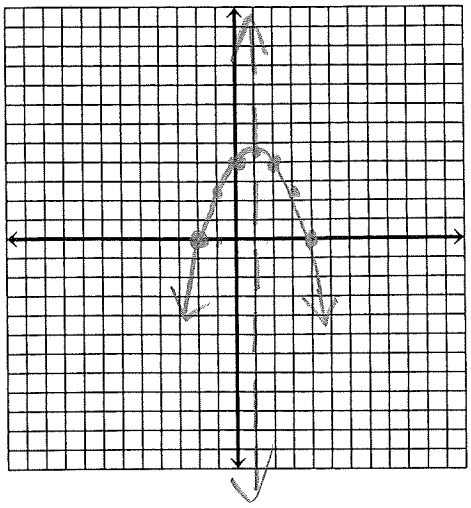
Up/Down: ↓ Axis of Symmetry: $x = 1$

Max or Min: Max Vertex: $-\frac{1}{2}(-3)(3)$ (1, 4.5)

Domain: all real numbers Range: $y \leq 4.5$

X intercepts: (4, 0), (−2, 0) (set y=0) Y intercept: (0, 4) (set x=0) $-\frac{1}{2}(-4)(2)$

x	y
-3	
5	
-1	$-\frac{1}{2}(-5)(1)$



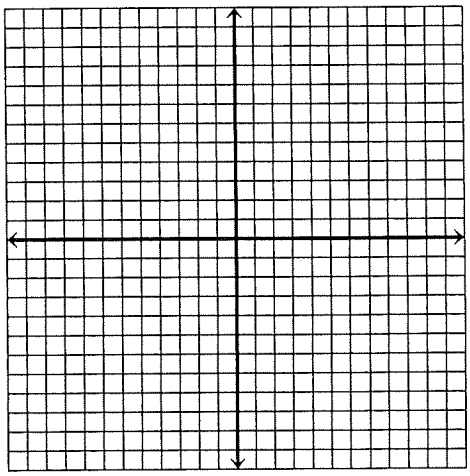
5. $f(x) = (x-2)(x+3)$

Up/Down: _____ Axis of Symmetry:_____

Max or Min:_____ Vertex:_____

Domain:_____ Range:_____

X intercepts:_____ (set y=0) Y intercept:_____ (set x=0)



6. $f(x) = -2(x+3)(x+1)$

Up/Down: _____ Axis of Symmetry:_____

Max or Min:_____ Vertex:_____

Domain:_____ Range:_____

X intercepts:_____ (set y=0) Y intercept:_____ (set x=0)