4-6 Completing the Square

Name:_____

Date:_____ Hour:_____

take note

Key Concept Completing the Square

You can turn the expression $x^2 + bx$ into a perfect square trinomial by adding $\left(\frac{b}{2}\right)^2$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Steps for solving by completing the square:

① Make sure the leading coefficient term "a" is ONE. If it is not, divide ***everything*** in the equation by the value of "a".
② Move all constants "c" to the right side of the equal sign.
③ Take half of "b" and square it and add it to **BOTH** sides of the equation.
④ Factor the left side, which should now be a perfect square trinomial.
⑤ Take the square root of each side, and create **two** "mini-equations" to solve.

Example 4 Solve a Perfect Square Trinomial Equation

A. $x^2 - 14x = 25$?

$x^2 - 14x + 49 = 25 + 49$

$\sqrt{(x-7)^2} = \sqrt{74}$

$x - 7 = \pm\sqrt{74}$

$x = 7 \pm \sqrt{74}$

B. $\frac{2x^2}{2} - \frac{12x}{2} - \frac{14}{2} = \frac{0}{2}$

$x^2 - 6x - 7 = 0$

$x^2 - 6x + 9 = 7 + 9$

$\sqrt{(x-3)^2} = \sqrt{16}$

$x - 3 = \pm 4$

$x = 3 + 4 = 7$

$x = 3 - 4 = -1$

C. $\frac{-3x^2}{-3} - \frac{12x}{-3} - \frac{9}{-3} = \frac{0}{-3}$

$x^2 + 4x + 3 = 0$

$x^2 + 4x + 4 = -3 + 4$

$\sqrt{(x+2)^2} = \sqrt{1}$

$x + 2 = \pm 1$

$x = -2 + 1 = -1$

$x = -2 - 1 = -3$

$x = -1$, $x = -3$

Homework: Solve each equation

19. $x^2 + 6x + 9 = 1$

21. $x^2 - 2x + 1 = 4$

35. $x^2 - 12x + 7 = 0$

37. $x^2 - 2x = 5$

39. $x^2 + 12 = 10x$

$x^2 - 10x = -12$

41. $x^2 + 2 = 6x + 4$

$x^2 - 6x = 2$

Steps for writing equations in vertex form:

① Factor the "a" value from the **first and second** terms – not from the constant. Move the constant off to the far right side of the equation.
② Complete the square in the parentheses.
③ Determine what quantity must be subtracted to "balance" the quantity used to complete the square.
④ Factor the perfect square binomial and add constants together to create an equation in vertex form.
⑤ Check your work using the formula $V_x = \frac{-b}{2a}$, if desired.

Example 5 Write the equation in vertex form by completing the square. Name the vertex and y intercept.

A. $y = x^2 + 8x - 6$

$x = \frac{-b}{2a} = \frac{-8}{2} = -4$

$y = (-4)^2 + 8(-4) - 6$

$y = 16 - 32 - 6$

$y = -16 - 6 = -22$

$y = 1(x+4)^2 - 22$

B. $y = 2x^2 - 6x - 8$

$x = \frac{-b}{2a} = \frac{6}{2(2)} = \frac{6}{4} = \frac{3}{2}$

$y = 2(\frac{3}{2})^2 - 6(\frac{3}{2}) - 8$

$2(\frac{9}{4}) - 9 - 8$

$4.5 - 9 - 8 = -12.5$

$(1.5, -12.5)$

-17.0
$+\ 4.5$
-12.5

Homework: Rewrite each equation in vertex form.

46. $y = x^2 + 4x + 1$

47. $y = 2x^2 - 8x + 1$

48. $y = -x^2 - 2x + 3$

49. $y = x^2 + 4x - 7$

50. $y = 2x^2 - 6x - 1$

51. $y = -x^2 + 4x - 1$

52. $f(x) = 3x^2 - 6x + 4$

53. $f(x) = -2x^2 - 8x + 6$

54. $f(x) = 4x^2 - 12x + 7$