Algebra 2 Name: Key

Section 4-7 – Notes and Examples Date: ________ Hour: ________

The Quadratic Formula

Consider the equation $3x^2+5x+1=0$. Try to solve this equation by factoring.

It's not possible, because the expression $3x^2+5x+1$ is not factorable. Completing the square is a method that will work for all quadratics (even those with complex solutions!), but it can get messy. (Go ahead – try this one...yuck!) So, long ago, mathematicians started with a quadratic function in standard form, and conducted the process of completing the square to find:

Here's how to solve $ax^2 + bx + c = 0$ to get the *Quadratic Formula.*

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$ Divide each side by a.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$ Rewrite so all terms containing x are on one side.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$ Complete the square.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$ Factor the perfect square trinomial. Also, simplify.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$ Find square roots.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$ Solve for x. Also, simplify the radical.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Simplify.

There are several methods to solve quadratic equations. We have discussed graphing to find the roots, factoring, using square roots, and completing the square. Graphing has limitations if the roots are not integers, and factoring is only useful if the expression in the equation is actually factorable (as discussed above). Square roots work best for simple quadratics, and completing the square can just get messy. The quadratic formula, as intimidating as it might look at first, is a method guaranteed to work on all quadratic equations (even those with complex solutions), and it's not as tedious as completing the square.

Take note **Key Concept The Quadratic Formula**

To solve the quadratic equation $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

And, yes, you need to KNOW this formula!

Before using the quadratic formula, be sure your equation is in Standard Form !!!

Example 1 – Find the zeros of each function by using the quadratic formula.

HW #11-21

a. $f(x) = \overset{a}{2}x^2 \overset{b}{-16}x + \overset{c}{27}$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{16 \pm \sqrt{(-16)^2 - 4(2)(27)}}{2 \cdot 2}$

$= \dfrac{16 \pm \sqrt{256-216}}{4}$

$= \dfrac{16 \pm \sqrt{40}}{4}$ $\qquad 40 = 4 \cdot 10,\ 4 = 2 \cdot 2$

$= \dfrac{\overset{8}{\cancel{16}} \pm \cancel{2}\sqrt{10}}{\cancel{4}\,2} = \dfrac{8 \pm \sqrt{10}}{2}$

b. $f(x) = 4x^2 + 3x + 2$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-3 \pm \sqrt{9-4(4)(2)}}{2(4)}$

$= \dfrac{-3 \pm \sqrt{9-32}}{8}$

$= \dfrac{-3 \pm \sqrt{-23}}{8}$

No Sol.

c. $f(x) = 3x^2 - 2 - x$

$y = 3x^2 - x - 2$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{1 \pm \sqrt{1-4(3)(-2)}}{2 \cdot 3}$

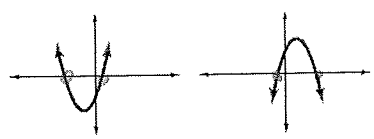
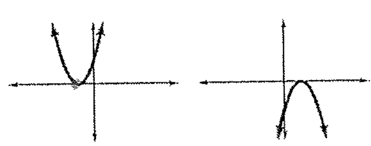
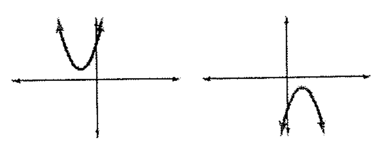
$= \dfrac{1 \pm \sqrt{25}}{6}$

$\dfrac{1 \pm 5}{6} \quad = \dfrac{1+5}{6} = 1$

$\dfrac{1-5}{6} = \dfrac{-4}{6} = -\dfrac{2}{3}$

Did you notice that the solutions to Example 1c were "nice," rational numbers (no square roots)? When this happens, it means that the expression in the equation WAS factorable! How can you tell before you begin? Use the ***discriminant***, which is the portion of the quadratic formula that is beneath the square root symbol ($b^2 - 4ac$). In the expression $3x^2 - x - 2$, the value of the discriminant would be $(-1)^2 - 4(3)(-2)$, or 25 (notice the discriminant does **not** include the square root itself). Since the discriminant is a perfect square, you would know the expression was (a) factorable, and (b) would yield two real solutions.

Discriminants and Solutions of Quadratic Equations

Value of the Discriminant	Number of Solutions for $ax^2 + bx + c = 0$	x-intercepts of Graph of Related Function $y = ax^2 + bx + c$
$b^2 - 4ac > 0$ positive	two real solutions	two x-intercepts 
$b^2 - 4ac = 0$	one real solution	one x-intercept 
$b^2 - 4ac < 0$	no real solutions	no x-intercepts 

Example 2 – Use the discriminant to determine the **quantity** and **type** of solution(s) for each quadratic equation.

$\boxed{\text{HW \#25-35}}$

a. $x^2 + 36 = 12x$

$x^2 - 12x + 36 = 0$

$b^2 - 4ac$

$(-12)^2 - 4(1)(36)$

$144 - 144 = 0$

1 Solution

b. $x^2 + 40 = 12x$

$x^2 - 12x + 40 = 0$

$b^2 - 4ac$

$(-12)^2 - 4(1)(40)$

$144 - 160 = -16$

No Sol.

c. $x^2 + 30 = 12x$

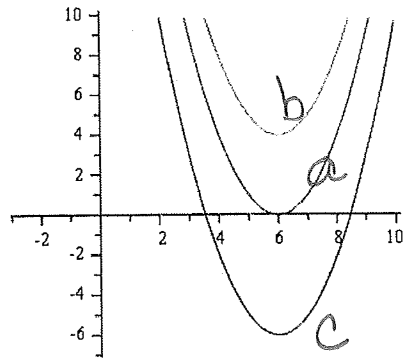
$x^2 - 12x + 30 = 0$

$(-12)^2 - 4(1)(30)$

$144 - 120 = 24$

2 Solutions

NOTE: The three equations above differ by only their constant value, c. Notice how similar their graphs look, but how different the results are in terms of the number of solutions!



The quadratic formula can be used to solve problems involving projectile motion.

$\boxed{\text{HW \#23,37}}$

Example 3 – A rocket is launched from the ground with an initial velocity of 150 feet per second. The function $h(t) = -16t^2 + 150t$ models the height in feet of the rocket at time t seconds.

When = t

a. Will the rocket reach a height of 300 feet? Yes

$h = -16t^2 + 150t$

$300 = -16t^2 + 150t$

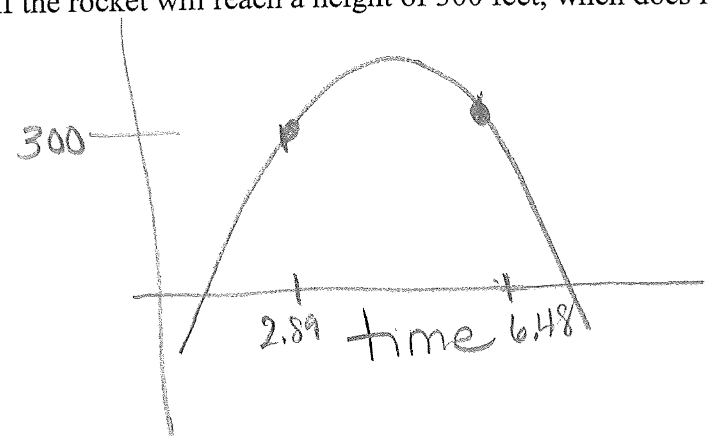
$0 = -16t^2 + 150t - 300$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-150 \pm \sqrt{150^2 - 4(-16)(-300)}}{2(-16)}$

$=$

b. If the rocket will reach a height of 300 feet, when does it occur?



$\frac{-150 \pm \sqrt{22{,}500 - 19{,}200}}{-32}$

$\frac{-150 \pm \sqrt{3{,}300}}{-32}$

$\frac{-150 \pm 57.44}{-32}$

$2.89, 6.48$

4.7 Homework. **Solve using the quadratic formula. Show your work. For solutions that are irrational, give both exact (radical) solutions AND decimal approximations rounded to the nearest hundredth.**

11. $x^2 - 4x + 3 = 0$

13. $2x^2 + 5x = 7$

15. $x^2 + 10x = -25$

17. $x^2 = 3x - 1$

19. $3x^2 = 2(2x + 1)$

21. $x(x - 5) = -4$

Evaluate the discriminant for each equation. Determine the number of of solutions.

25. $x^2 + 4x + 5 = 0$

27. $-4x^2 + 20x - 25 = 0$

29. $2x^2 + 7x - 15 = 0$

31. $-2x^2 + 7x = 6$

23. Your class is selling boxes of flower seeds as a fundraiser. The total profit, p, depends on the amount x, that your class charges for each box of seeds. The equation $p = -.5x^2 + 25x - 150$ models the profit of the fundraiser. What's the smallest amount, in dollars, that you can charge and make a profit of at least \$125?