Name:______________________________ Date:____________________ Hour:___________

3.4 Graphs and Transformations

Parent Functions

The functions on the next page are often called **parent functions.** A parent function is a function with a certain shape that has the simplest algebraic rule for that shape. For example, $f(x) = x^2$ is the simplest rule for a parabola. You should memorize the basic shapes of the parent functions.

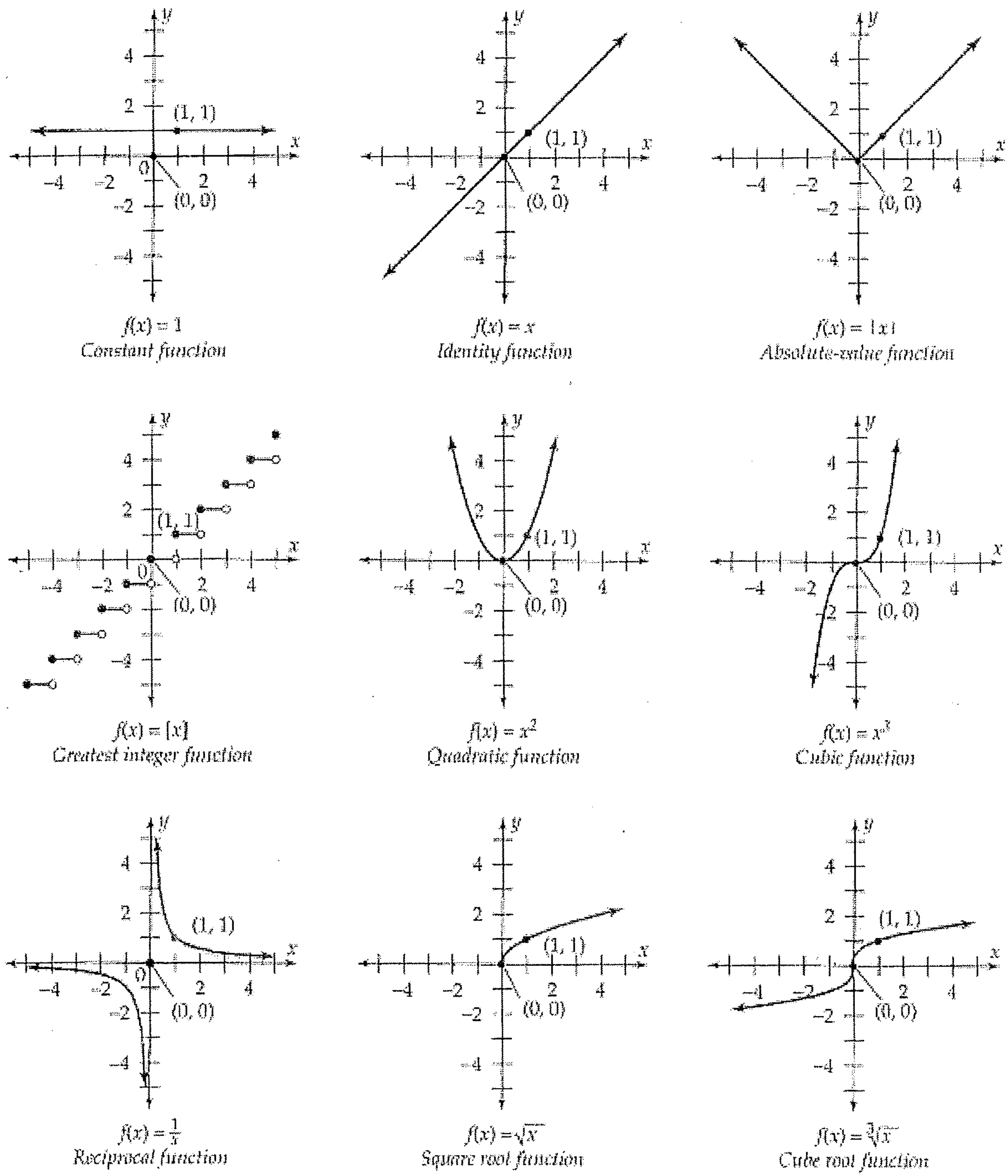


Figure 3.4-1

The parent functions will be used to illustrate the rules for the basic transformations. Remember, however, that these transformation rules work for *all* functions.

Vertical Shifts

Let c be a positive number.

The graph of $g(x) = f(x) + c$ is the graph of f shifted upward c units.

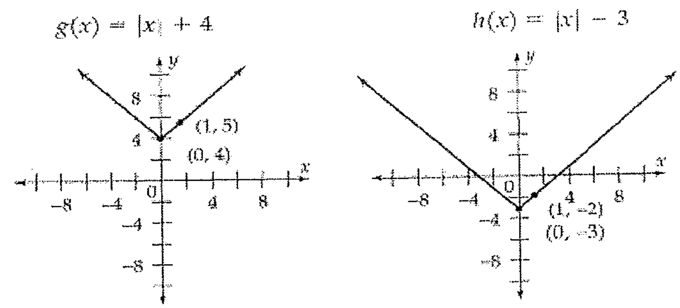
The graph of $g(x) = f(x) - c$ is the graph of f shifted downward c units.

Example 1 Shifting a Graph Vertically

Graph $g(x) = |x| + 4$ and $h(x) = |x| - 3$.

Solution

The parent function is $f(x) = |x|$. The graph of $g(x)$ is the graph of $f(x) = |x|$ shifted upward 4 units, and the graph of $h(x)$ is the graph of $f(x) = |x|$ shifted downward 3 units.



Horizontal Shifts

Let c be a positive number.

The graph of $g(x) = f(x + c)$ is the graph of f shifted c units to the left.

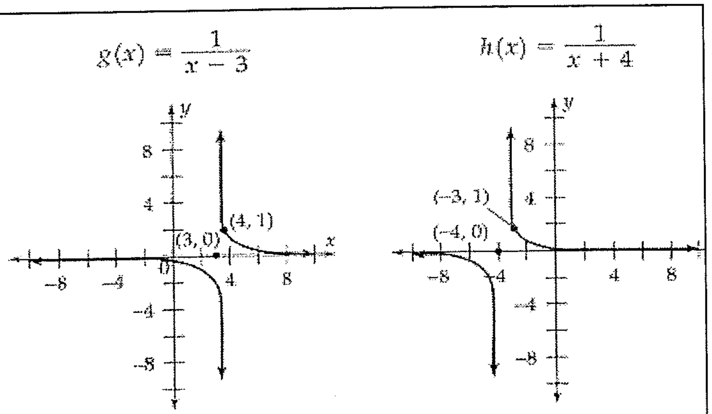
The graph of $g(x) = f(x - c)$ is the graph of f shifted c units to the right.

Example 2 Shifting a Graph Horizontally

Graph $g(x) = \frac{1}{x - 3}$ and $h(x) = \frac{1}{x + 4}$.

Solution

The parent function is $f(x) = \frac{1}{x}$. The graph of $g(x)$ is the graph of $f(x) = \frac{1}{x}$ shifted 3 units to the right and the graph of $h(x)$ is the graph of $f(x) = \frac{1}{x}$ shifted 4 units to the left.



Reflections

The graph of $g(x) = -f(x)$ is the graph of f reflected across the x-axis.

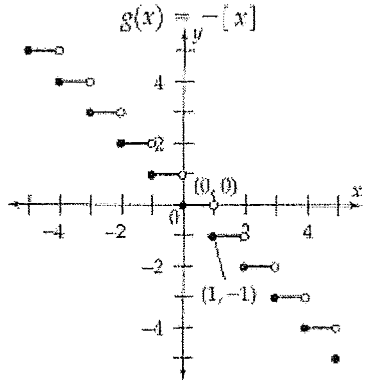
The graph of $g(x) = f(-x)$ is the graph of f reflected across the y-axis.

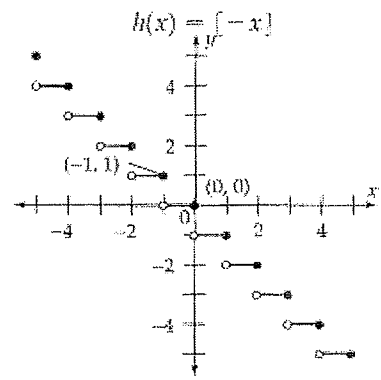
Example 3 Reflecting a Graph Across the x- or y-Axis

Graph $g(x) = -[x]$ and $h(x) = [-x]$.

Solution

The parent function is $f(x) = [x]$. The graph of $g(x)$ is the graph of $f(x) = [x]$ reflected across the x-axis, and the graph of $h(x)$ is the graph of $f(x) = [x]$ reflected across the y-axis. One difference in the reflections is whether the endpoint of each segment is included on the left or on the right. Study the functions closely, along with the parent function, to determine another difference in the two reflections.





Stretches and Compressions

Let c be a positive number.

Vertical Stretches and Compressions

If (x, y) is a point on the graph of $f(x)$, then (x, cy) is a point on the graph of $g(x) = c \cdot f(x)$.

If $c > 1$, the graph of $g(x) = c \cdot f(x)$ is the graph of f stretched vertically, away from the x-axis, by a factor of c.

If $c < 1$, the graph of $g(x) = c \cdot f(x)$ is the graph of f compressed vertically, toward the x-axis, by a factor of c.

Horizontal Stretches and Compressions

If (x, y) is a point on the graph of $f(x)$, then $\left(\frac{1}{c}x, y\right)$ is a point on the graph of $g(x) = f(c \cdot x)$.

If $c > 1$, the graph of $g(x) = f(c \cdot x)$ is the graph of f compressed horizontally, toward the y-axis, by a factor of $\frac{1}{c}$.

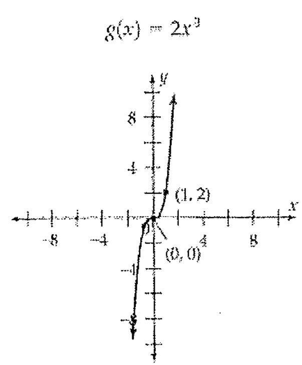
If $c < 1$, the graph of $g(x) = f(c \cdot x)$ is the graph of f stretched horizontally, away from the y-axis, by a factor of $\frac{1}{c}$.

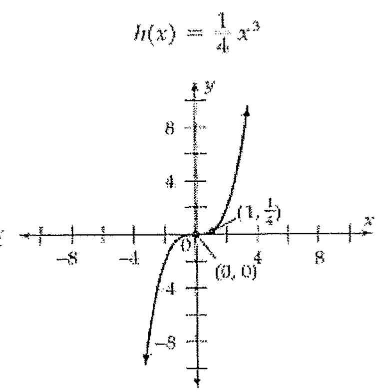
Example 4 Vertically Stretching and Compressing a Graph

Graph $g(x) = 2x^3$ and $h(x) = \frac{1}{4}x^3$.

Solution

The parent function is $f(x) = x^3$. For the function $g(x) = 2x^3$, every y-coordinate of the parent function is multiplied by 2, stretching the graph of the function in the vertical direction, away from the x-axis. For the function $h(x) = \frac{1}{4}x^3$, every y-coordinate of the parent function is multiplied by $\frac{1}{4}$, compressing the graph of the function in the vertical direction, toward the x-axis.





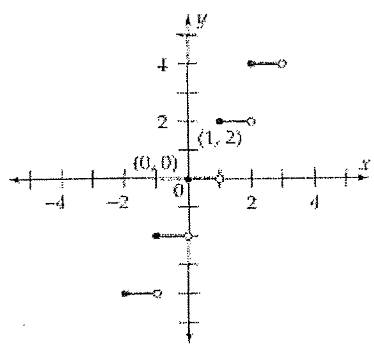
Example 5 Stretching a Function Vertically and Horizontally

Graph $g(x) = 2[x]$ and $h(x) = [2x]$.

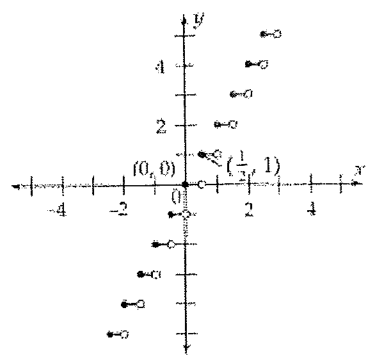
Solution

The parent function is $f(x) = [x]$. In $g(x) = 2[x]$, the y-values are multiplied by 2, and in $h(x) = [2x]$, the x-values are multiplied by 2. The result is that in the graph of the first function, the "steps" get higher, and in the graph of the second function, they get narrower.

$g(x) = 2[x]$



$h(x) = [2x]$



Combining Transformations

For a function of the form $g(x) = c \cdot f(a(x - b)) + d$, first graph $f(x)$.

1. If $a < 0$, reflect the graph across the y-axis.
2. Stretch or compress the graph horizontally by a factor of $\left|\frac{1}{a}\right|$.
3. Shift the graph horizontally by b units: right if $b > 0$, and left if $b < 0$.
4. If $c < 0$, reflect the graph across the x-axis.
5. Stretch or compress the graph vertically by a factor of $|c|$.
6. Shift the graph vertically by d units: up if $d > 0$, and down if $d < 0$.

Example 6 Combining Transformations. Graph the following.

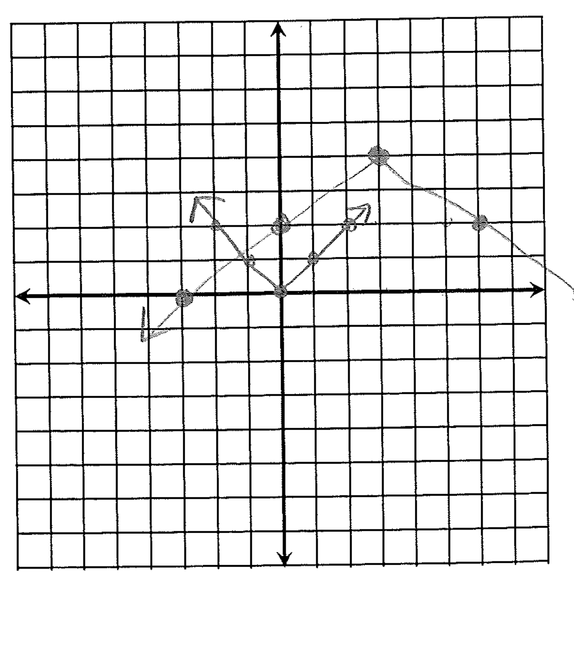
$y = |x|$

$f(x) = -2\left|\frac{1}{3}x - 1\right| + 4$

$-2\left|\frac{1}{3}(x-3)\right| + 4$

Parent | a= | b= | c= | d=

$y=\|x\|$		mult x by 3		add 3 to x		mult y (-2)		add 4 to y	
x	y	x	y	x	y	x	y	x	y
-2	2	-6	2	-3	2	-3	-4	-3	0
-1	1	-3	1	0	1	0	-2	0	2
0	0	0	0	3	0	3	0	3	4
1	1	3	1	6	1	6	-2	6	2
2	2	6	2	9	2	9	-4	9	0



Precalculus

3.4 Transformations

NO CALCULATOR!

Name: ______________________

Date: ______________ Hour: ______

I. (a) Identify the parent function that could be used to graph the function. (b) Describe the sequence of transformations that transform the graph of the parent function into the graph of $g(x)$. **List the transformations in the proper order.**

1. $g(x) = -2(x-3)^2 + 2$

a. ______________________

b. ______________________

____

____

____

2. $g(x) = \frac{2}{3}|6-2x| - 2$

a. ______________________

b. ______________________

____

____

____

3. $g(x) = -\frac{1}{4}(2x-4)^2 - 1$

a. ______________________

b. ______________________

____

____

____

4. $g(x) = -4(\frac{1}{2}x-3)^3 + 2$

a. ______________________

b. ______________________

____

____

____

5. $g(x)=-\sqrt[3]{\frac{1}{2}x-4}$

a. ______________________________

b. ______________________________

____

____

____

6. $g(x)=-2\left[\!\left[\frac{1}{3}x-\frac{1}{2}\right]\!\right]+3$

a. ______________________________

b. ______________________________

____

____

____

7. $g(x)=3(-4x+8)^3$

a. ______________________________

b. ______________________________

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____

____

8. $g(x)=-5\sqrt{\frac{1}{3}x-2}+2$

a. ______________________________

b. ______________________________

____

____

____

9. $g(x)=-2\left|\frac{3}{5}x-4\right|-2$

a. ______________________________

b. ______________________________

____

____

II. Write a rule (an equation) for the function whose graph could be obtained by performing the given transformations.

10. Parent function: $f(x) = x^2$ 10. ____________________

Transformations:
- Shift the graph 1 units to the left
- Stretch it vertically by a factor of 4
- Shift it downward 5 units
- Reflect over x axis

11. Parent function: $f(x) = x^3$ 11. ____________________

Transformations:
- Shift the graph 2 units to the left
- Stretch it horizontally by a factor of 3
- Shift it upward 5 units
- Reflect over y axis

12. Parent function: $f(x) = |x|$ 12. ____________________

Transformations:
- Shift the graph 2 units to the right
- Stretch it vertically by a factor of 3
- Shift it upward 5 units
- Compress horizontally by ½
- Reflect over x axis

13. Parent function: $f(x) = \sqrt[3]{x}$ 13. ____________________

Transformations:
- Shift the graph 4 units up
- Stretch it vertically by a factor of 3
- Shift it left 5 units
- Reflect over y axis

14. Parent function: $f(x) = \frac{1}{x}$ 14. ____________________

Transformations:
- Shift the graph 2 units to the right
- Compress it horizontally by a factor of 1/3
- Shift it downward 5 units
- Reflect over y axis

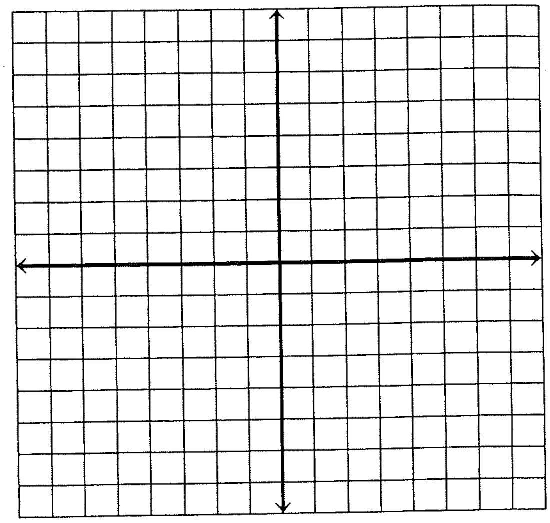
15. Parent function: $f(x) = \frac{1}{x}$ 15. ____________________

Transformations:
- Shift the graph 2 units up
- Vertical stretch by 3
- Shift it left 5 units
- Reflect over x axis

For Exercises 16-19, complete the table, and graph each function on the grid provided.

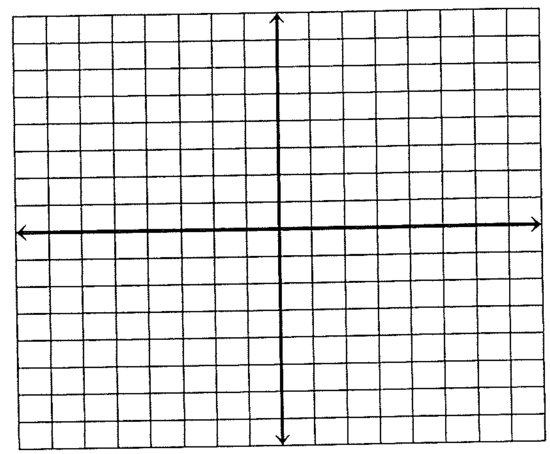
16. $y=-2\sqrt{\frac{1}{2}x-1}+3$

Type:				
x	y			



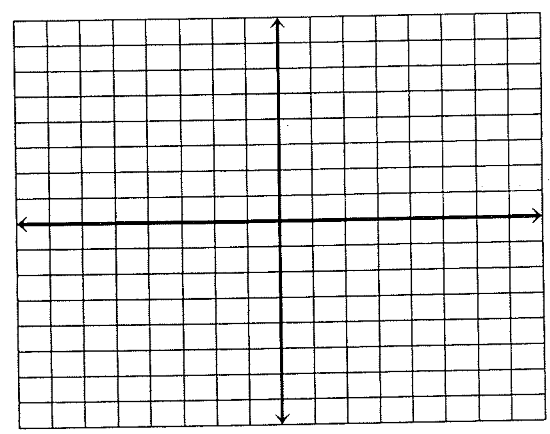
17. $y=3\left|-x+4\right|-3$

Type:				
x	y			



18. $y=\frac{1}{2}(-\frac{1}{2}x-1)^2+3$

Type:				
x	y			



19. $y=\sqrt[3]{-2x-4}+2$

Type:				
x	y