Algebra 2

Section 4-8 – Notes and Examples

Name: ______________________

Date: ________________ Hour: _______

Complex Numbers – Day 2

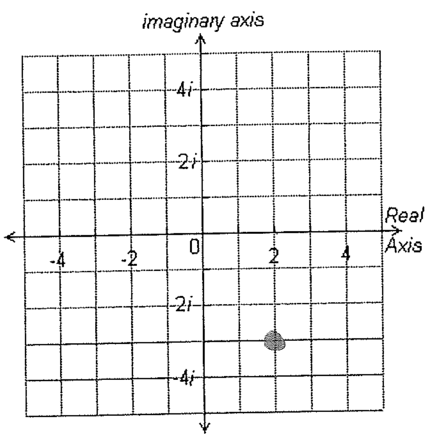
The complex plane is a set of coordinate axes in which the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.

$6i - 2$

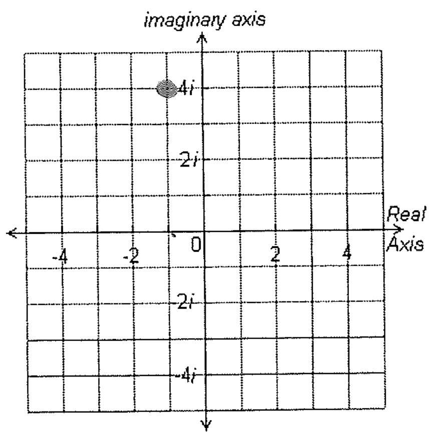
$-2 + 6i$

Example 1 – Graph each complex number.

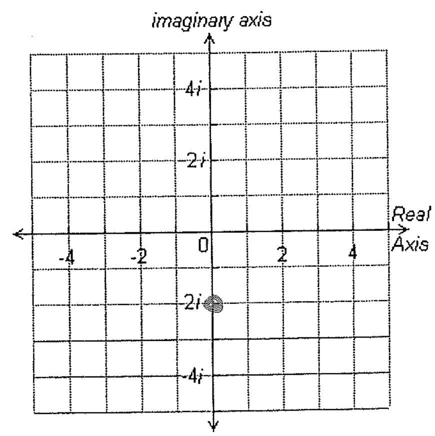
a. $2 - 3i$



b. $-1 + 4i$

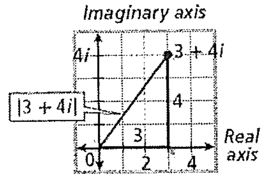


c. $-2i$

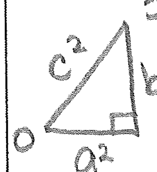


You know that the absolute value of a real number is its distance from 0 on the real axis along a number line. In the same way, the absolute value of an imaginary number is its distance from 0 along the imaginary axis. You can use the Pythagorean Theorem to calculate the distance.

Absolute Value of a Complex Number

WORDS	ALGEBRA	EXAMPLE
The absolute value of a complex number $a + bi$ is the distance from the origin to the point (a, b) in the complex plane, and is denoted $\lvert a + bi \rvert$.	$\lvert a + bi \rvert = \sqrt{a^2 + b^2}$	Imaginary axis  $\lvert 3 + 4i \rvert = \sqrt{3^2 + 4^2}$ $= \sqrt{9 + 16}$ $= 5$

$3+4i$



$3^2 + 4^2 = c^2$

$9 + 16 = c^2$

$25 = c^2$

$5 = c$

Example 2 – Find each absolute value.

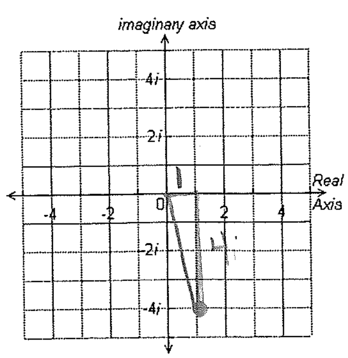
a. $\lvert 1 - 4i \rvert$

$1^2 + 4^2 = c^2$

$1 + 16 = c^2$

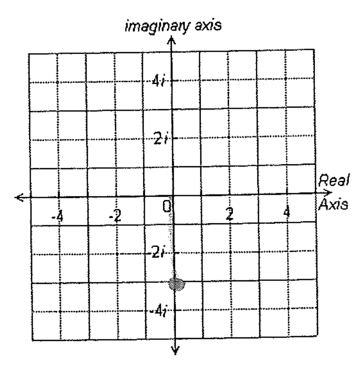
$\sqrt{17} = \sqrt{c^2}$

$\sqrt{17} = c$



b. $\lvert -3i \rvert$

3



A quadratic function whose graph does not cross the x-axis has no real solutions. However, every quadratic function has complex solutions, which utilize ***imaginary numbers***.

Example 3– Solve each equation.

a. $5x^2 + 90 = 0$

$$\frac{5x^2}{5} = \frac{-90}{5}$$

$$\sqrt{x^2} = \sqrt{-18}$$

$$x = \pm i\sqrt{18}$$

$$x = \pm 3i\sqrt{2}$$

$$= \pm 3\sqrt{2}\, i$$

18 → 2, 9; 9 → 3, 3

b. $-7x^2 - 5 = 0$

$$\frac{-7x^2}{-7} = \frac{5}{-7}$$

$$\sqrt{x^2} = \sqrt{-\frac{5}{7}}$$

$$x = \pm i\sqrt{\frac{5}{7}}$$

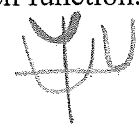
$$x = \pm \frac{\sqrt{35}}{7} i$$

$$\frac{\sqrt{5}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

You can still use the process of completing the square or the quadratic formula to find solutions to quadratic equations, even if those solutions are imaginary or complex.

Example 4 – Find the zeros of each function.

$y = x^2 + 10x + 26$



a. $f(x) = x^2 + 10x + 26$

$$0 = x^2 + 10x + 26$$

$$x^2 + 10x + 26 = 0$$

$$x^2 + 10x + 25 = -26 + 25$$

$$\sqrt{(x+5)^2} = \sqrt{-1}$$

$$x + 5 = \pm i$$

$$x = -5 \pm i$$

b. $f(x) = x^2 + 4x + 12$

$$x^2 + 4x + 12 = 0$$

$$x^2 + 4x + 4 = -12 + 4$$

$$\sqrt{(x+2)^2} = \sqrt{-8}$$

$$x + 2 = \pm 2\sqrt{2}\, i$$

$$x = -2 \pm 2\sqrt{2}\, i$$

Two complex numbers are equal if and only if their real parts are equal and their complex parts are equal.

Example 5 – Find the values of x and y that make the equation $\underbrace{4x}_{a} + \underbrace{10i}_{bi} = \underbrace{2}_{a} - \underbrace{(4y)i}_{bi}$ true.

Set $a = a$

$$4x = 2$$

$$x = 2/4 = 1/2$$

Set $bi = bi$

$$10\cancel{i} = -4y\cancel{i}$$

$$\frac{10}{-4} = \frac{-4y}{-4}$$

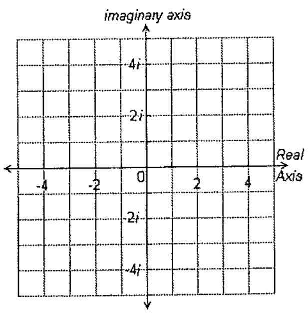
$$-5/2 = y$$

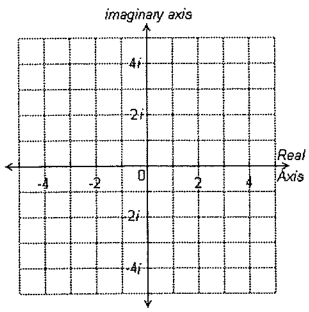
4.8 Algebra 2
Homework Day 2

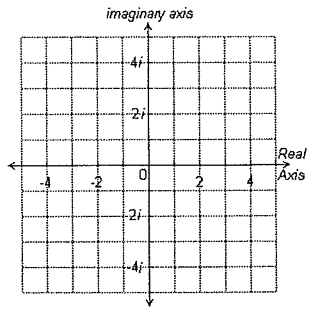
Name____________________

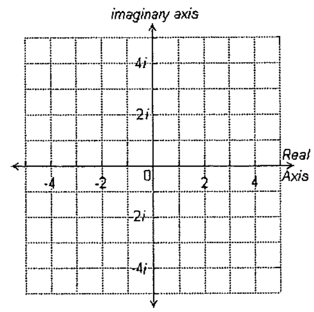
Plot each complex number and find its absolute value.

13. $2i$ **14.** $5 + 12i$ **15.** $2 - 2i$ **16.** $1 - 4i$









Solve each equation. Check your answer.

See Probl

33. $x^2 + 25 = 0$ **34.** $2x^2 + 1 = 0$ **35.** $3s^2 + 2 = -62$

36. $x^2 = -7$ **37.** $x^2 + 36 = 0$ **38.** $-5x^2 - 3 = 0$

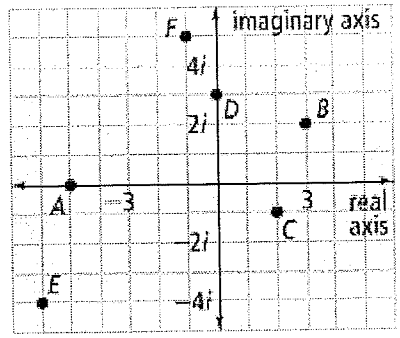
Find all solutions to each quadratic equation.

See Probl

39. $x^2 + 2x + 3 = 0$ **40.** $-3x^2 + x - 3 = 0$ **41.** $2x^2 - 4x + 7 = 0$

42. $x^2 - 2x + 2 = 0$ **43.** $x^2 + 5 = 4x$ **44.** $2x(x - 3) = -5$

45. a. Name the complex number represented by each point on the graph at the right.
b. Find the additive inverse of each number.
c. Find the complex conjugate of each number.
d. Find the absolute value of each number.



47. Solve $(x + 3i)(x - 3i) = 34$.

Simplify each expression.

48. $(8i)(4i)(-9i)$

49. $(2 + \sqrt{-1}) + (-3 + \sqrt{-16})$

50. $(4 + \sqrt{-9}) + (6 - \sqrt{-49})$

51. $(10 + \sqrt{-9}) - (2 + \sqrt{-25})$

52. $(8 - \sqrt{-1}) - (-3 + \sqrt{-16})$

53. $2i(5 - 3i)$

54. $-5(1 + 2i) + 3i(3 - 4i)$

55. $(3 + \sqrt{-4})(4 + \sqrt{-1})$

Two complex numbers $a + bi$ and $c + di$ are equal when $a = c$ and $b = d$. Solve each equation for x and y.

67. $2x + 3yi = -14 + 9i$

68. $3x + 19i = 16 - 8yi$

69. $-14 - 3i = 2x + yi$

73. How can you rewrite the expression $(8 - 5i)^2$ in the form $a + bi$?

(A) $39 + 80i$ (B) $39 - 80i$ (C) $69 + 80i$ (D) $69 - 80i$