Algebra 2

Section 5.1 – Notes and Examples

Name:_________________________________

Date:_____________________ Hour: ______

Polynomial Functions

Definitions/examples:

A ***monomial*** is a real number, a variable, or a product of a real number and one or more variables with whole number exponents.

The ***degree of a of a monomial*** in one variable is the ***exponent*** of the variable.

A ***polynomial*** is a monomial or a sum of monomials.

The ***degree of a polynomial*** in one variable is the ***greatest degree exponent*** among its monomial terms.

Polynomials should be written in *standard form*:

You also should know that the "4" is called the *leading coefficient.*

take note

Key Concept Standard Form of a Polynomial Function

The **standard form of a polynomial function** arranges the terms by degree in descending numerical order.

$$P(x) = 4x^3 + 3x^2 + 5x - 2$$

Cubic term | Quadratic term | Linear term | Constant term

Example 1 – Write the polynomial $3x - 4x^3 + 2x^2 + 10$ in standard form, state the number of terms, and identify what type of term each is.

$-4x^3 + 2x^2 + 3x + 10$

Polynomials are grouped in two ways, so every polynomial will have two "names" based on:

❶ The degree of the polynomial

❷ The number of terms

Polynomial Example	Degree	Name Using Degree	Number of Terms	Name Using Number of Terms
5	0	Constant	1	monomial
$x+4$	1	linear	2	binomial
$4x^2$	2	quadratic	1	monomial
$4x^3 - 2x^2 + x$	3	Cubic	3	trinomial
$2x^4 + 5x^2$	4	quartic	2	binomial
$-x^5 + 4x^2 + 2x + 1$	5	quintic	4	polynomial w/4 terms

Example 2 – Write each polynomial in standard form. What is the classification of each polynomial by degree? By number of terms?

a. $3x^3 - x + 5x^4$ quartic trinomial

$5x^4 + 3x^3 - x$

b. $3 - 4x^5 + 2x^2 + 10$ quintic trinomial

$-4x^5 + 2x^2 + 13$

It is important to identify the degree of a polynomial because the degree will affect the shape of a graph and tell you several things.

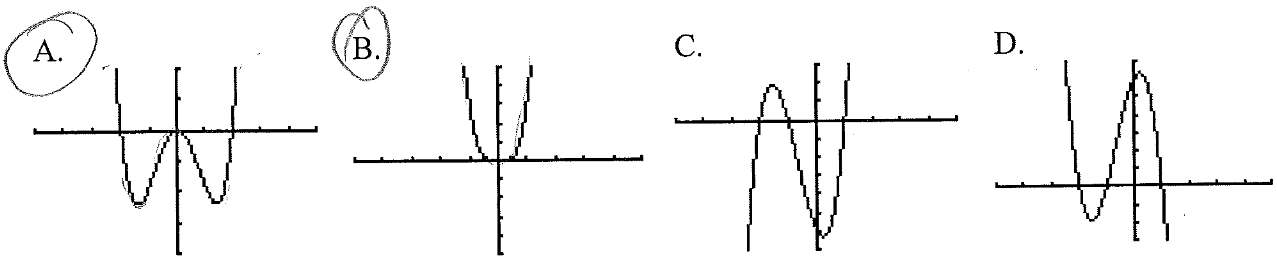
First, the degree will tell you what the ***end behavior*** of a graph will be. End behavior refers to what the graph does at the far left and far right side of the graph. Does the function go up forever or down forever in each direction?

Example 3 – Without graphing, describe the end behavior for each polynomial function (left then right).

a. $f(x) = 4x - 6x^2 + 1$ b. $f(x) = 3x^7 + 5x + 10$ c. $f(x) = -14x^3 - 6x^2 - 2x$

$-6x^2 + 4x + 1$

Example 4 – Determine (a) the sign (+ or –) of the leading coefficient, and (b) the smallest possible degree for each polynomial graph. Which of the graphs below ***could*** be the graph of a polynomial whose leading term is $3x^4$?



The degree also will tell you the ***maximum*** number of ***turning points*** in a graph.

The number of possible turning points is always ***one less*** than the ***degree of the polynomial***. There could be less!

Refer to Example 4. There were TWO possible answers for $3x^4$ when we considered end behavior. One of the examples had 3 turning points, while the other had only 1 turning point, both of which are less than the degree of 4. This tells us that at most the number of turning points is the degree -1 or *n-1*.

The graph of a polynomial function of ***odd degree*** has an ***even*** number of turning points. The graph of a polynomial function of ***even degree*** has an ***odd*** number of turning points.

Example 5 – Without graphing, state the maximum number of turning points for each polynomial function and list the possible number of turning points.

a. $f(x) = 4x - 6x^2 + 1$

$-6x^2 + 4x + 1$

Maximum T.P. 1

Possible T.P. 1

b. $f(x) = 3x^7 + 5x + 10$

Maximum T.P. 6

Possible T.P. 6, 4, 2

c. $f(x) = -14x^4 - 6x^2 - 2x$

Maximum T.P. 3

Possible T.P. 1, 3

Polynomial Functions – Common Differences

If you have a table of values from a polynomial function, you can determine the degree of that polynomial using ***common differences***. You must:

❶ Confirm that the x-values change by a common difference.
❷ Compare the differences between the y-values until the value is the same.

Example 6 – The table of values represents ordered pairs from a **linear** function (degree of 1). Use common differences to confirm.

x	y
–2	–8
–1	–5
0	–2
1	1
2	4

+3 +3 +3 +3

Example 7 – Each table of values represents ordered pairs from **polynomial** functions – but NOT linear functions! Use common differences to determine the degree of the polynomial.

a.

x	y
–2	–13
–1	–4
0	–1
1	2
2	11
3	32
4	71

+9 +3 +3 +9 +21 +39

–6 0 +6 +12 +18

+6 +6 +6 +6

Cubic

b.

x	y
–3	23
–2	–16
–1	–15
0	–10
1	–13
2	–12
3	29

23 –16 –15 –10 –13 –12 29

–39 +1 +5 –3 +1 +41 linear

+40 +4 –8 4 +40 quadratic

–36 –12 +12 +36 cubic

+24 +24 +24 quartic

Classify each polynomial by degree and by number of terms. Simplify first if necessary.

$4x^5 - 5x^2 + 3 - 2x^2$

$4x^5 - 7x^2 + 3$ quintic trinomial

$(7x^2 + 9x - 5) + (9x^2 - 9x)$

$16x^2 - 5$ quadratic binomial

$(4s^4 - s^2 - 3) + (3s + s^2 + 5)$

$4s^4 - 3s + 2$

quartic trinomial

Assignment: Lesson 5.1, Practice 5.1

5-1 Practice

Form G

Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. $4x + x + 2$
2. $-3 + 3x - 3x$
3. $6x^4 - 1$
4. $1 - 2s + 5s^4$
5. $5m^2 - 3m^2$
6. $x^2 + 3x - 4x^3$
7. $-1 + 2x^2$
8. $5m^2 - 3m^3$
9. $5x - 7x^2$
10. $2 + 3x^3 - 2$
11. $6 - 2x^3 - 4 + x^3$
12. $6x - 7x$
13. $a^3(a^2 + a + 1)$
14. $x(x + 5) - 5(x + 5)$
15. $p(p - 5) + 6$

Determine the end behavior of the graph of each polynomial function.

22. $y = 3x^4 + 6x^3 - x^2 + 12$
23. $y = 50 - 3x^3 + 5x^2$
24. $y = -x + x^2 + 2$
25. $y = 4x^2 + 9 - 5x^4 - x^3$
26. $y = 12x^4 - x + 3x^7 - 1$
27. $y = 2x^5 + x^2 - 4$

Describe the shape of the graph of each cubic function by determining the end behavior and number of turning points.

31. $y = x^3 + 4x$
32. $y = -2x^3 + 3x - 1$
33. $y = 5x^3 + 6x^2$

Determine the degree of the polynomial function with the given data.

34.

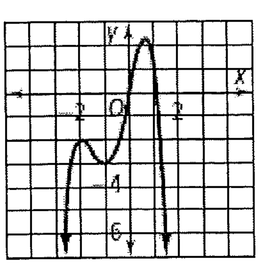
x	y
−2	−16
−1	1
0	4
1	5
2	16

35.

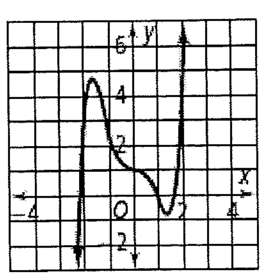
x	y
−2	52
−1	6
0	2
1	4
2	48

Determine the sign of the leading coefficient and the degree of the polynomial function for each graph.

36.



37.



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