3.1 Functions

Definition of a Function

A *function* consists of

- a set of inputs, called the *domain*
- a rule by which each input determines one and only one output
- a set of outputs, called the *range*

The phrase "one and only one" means that for each input (element of the domain), the rule of a function determines exactly one output (element of the range). However, different inputs may produce the same output.

Example 2 Determining Whether a Relation is a Function

The tables below list the inputs and outputs for two relations. Determine whether each relation is a function.

a.

Inputs	1	1	2	3	3
Outputs	5	6	7	8	9

b.

Inputs	1	3	5	7	9
Outputs	5	5	7	8	5

a. Yes or (no)

b. (Yes) or no

Example 5 Determining if an Equation Defines a Function

Is an equation a function? Solve the equation for y and if it is unique (no +/-), then YES!!!!

Determine whether each equation defines y as a function of x.

a. $4x - 2y^3 + 5 = 0$

b. $y^2 - x + 1 = 0$

Like 5-12

a) $4x - 2y^3 + 5 = 0$

$-2y^3 = -4x - 5$

$\frac{-2y^3}{-2} = \frac{-4x}{-2} - \frac{5}{-2}$

$\sqrt[3]{y^3} = \sqrt[3]{2x + 2.5}$

$y = \sqrt[3]{2x + 2.5}$

$y^2 - x + 1 = 0$

$\sqrt{y^2} = \sqrt{x - 1}$

$y = \pm\sqrt{x - 1}$

NOT

Example 3 Evaluating a Function

Like 13-33

Find the indicated values of $f(x) = \sqrt{x^2 + 1}$.

a. $f(3)$ b. $f(-5)$ c. f(p) d. f(p+1) e. f(x+h)

$f(3) = \sqrt{x^2+1}$

$\sqrt{3^2+1}$

$\sqrt{10}$

$\sqrt{(-5)^2+1}$

$\sqrt{26}$

$\sqrt{p^2+1}$

$\sqrt{(p+1)^2+1}$

$\sqrt{p^2+2p+1+1}$

$\sqrt{p^2+2p+2}$

$\sqrt{(x+h)^2+1}$

$\sqrt{x^2+2xh+h^2+1}$

Example 4 The difference quotient: $\frac{f(x+h)-f(x)}{h}$

Like 35-42

Use the difference quotient to evaluate $f(x)=x^2-x+2$

$f(x+h)=(x+h)^2-(x+h)+2$

$=(x^2+2xh+h^2)-x-h+2$

$f(x+h)=[x^2+2xh+h^2-x-h+2]$

$f(x)=[x^2-x+2]$

$$\frac{[x^2+2xh+h^2-x-h+2]-[x^2-x+2]}{h}$$

$$\frac{x^2+2xh+h^2-x-h+2-x^2+x-2}{h}$$

$$\frac{2xh+h^2-h}{h} \rightarrow \frac{h(2x+h-1)}{h}$$

Like 43-56

Determining Domains

To determine what the domain of a function is, it's best to look at places in the function where it is undefined. In other words…what it CAN'T BE!! We will use interval notation to describe the domains.

A. $y=\sqrt[3]{2x+5}$

$\sqrt[3]{8}$

$\sqrt[3]{-8}$

$(-\infty,\infty)$

B. $y=\sqrt{x+2}$ ← must be (+)

$x+2\geq 0$

$-2 \quad -2$

$x\geq -2$

-2

d: $[-2,\infty)$

$y=\frac{x^2-9}{x-1}$

$x-1=0$

$x=1$

1

d: $(-\infty,1)\cup(1,\infty)$

Like 57-62

Evaluating a piecewise function

Given: $f(x)=\begin{cases}2x+3, & x<4\\ x^2-1, & 4\leq x\leq 10\end{cases}$

4

4 10

$f(4)=$ 4^2-1

$16-1=15$

$f(10)=$ 99

Find the domain: $(-\infty,10]$

$f(3)=9$

Greatest integer function: For any number x, round down to the nearest integer. (Always go to the left of the number line). You are rounding down.

$f(x)=[x]$

$f(-4.1)=$ $[-4.1]=5$

$f(4.9)=$ $[4.9]=4$

$f(-2.9)=$ -3

$f(10.9)=$ 10

-4 -3

4 5

PreCalc
Homework 3.1

Name_____

In Exercises 5–12, determine whether the equation defines y as a function of x.

5. $y = 3x^2 - 12$

7. $y^2 = 4x + 1$

9. $3x + 2y = 12$

11. $x^2 + y^2 = 9$

Exercises 13–34 refer to the functions below. Find the indicated value of the function.

$$f(x) = \sqrt{x + 3} - x + 1$$
$$g(t) = t^2 - 1$$
$$h(x) = x^2 + \frac{1}{x} + 2$$

13. $f(0)$

15. $f(\sqrt{2})$

17. $f(-2)$

19. $h(3)$

21. $h\left(\frac{3}{2}\right)$

23. $h(a + k)$

25. $h(2 - x)$

27. $g(3)$

29. $g(0)$

31. $g(s + 1)$

33. $g(-t)$

In Exercises 35–42, compute and simplify the difference quotient (shown below). Assume $h \neq 0$.

$$\frac{f(x + h) - f(x)}{h}$$

35. $f(x) = x + 1$

37. $f(x) = 3x + 7$

39. $f(x) = x - x^2$

In Exercises 43–56, determine the domain of the function according to the domain convention.

43. $f(x) = x^2$

44. $g(x) = \frac{1}{x^2} + 2$

45. $h(t) = |t| - 1$

46. $k(u) = \sqrt{u}$

47. $k(x) = |x| + \sqrt{x} - 1$

48. $h(x) = \sqrt{(x+1)^2}$

49. $g(u) = \frac{|u|}{u}$

50. $h(x) = \frac{\sqrt{x-1}}{x^2 - 1}$

51. $g(y) = [-y]$

52. $f(t) = \sqrt{-t}$

53. $g(u) = \frac{u^2 + 1}{u^2 - u - 6}$

54. $f(t) = \sqrt{4 - t^2}$

In Exercises 57–62, find the following:

a. $f(0)$
b. $f(1.6)$
c. $f(-2.3)$
d. $f(5 - 2\pi)$
e. The domain of f

57. $f(x) = [x]$

59. $f(x) = \begin{cases} x^2 + 2x & \text{if } x < 2 \\ 3x - 5 & \text{if } 2 \leq x \leq 20 \end{cases}$

61. $f(x) = \begin{cases} 2x - 3 & \text{if } x < -1 \\ |x| - 5 & \text{if } -1 \leq x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$