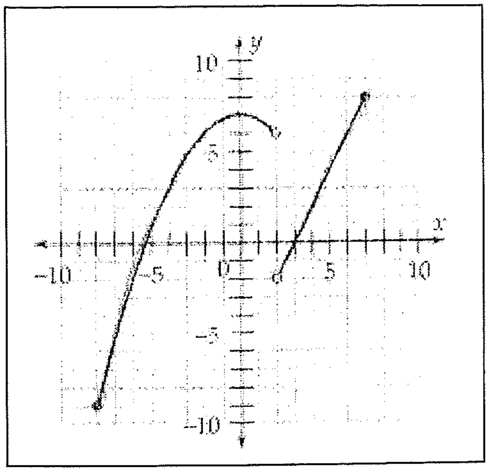
3.2 Graphs of Functions

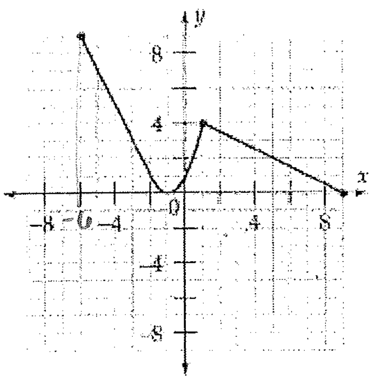
Example 1 A Function Defined by a Graph

The graph in Figure 3.2.1 defines the function f. Determine the following.



a. $f(0)$ 7 **b.** $f(3)$ 0 **c.** the domain of f $[-8,2)\cup(2,7]$ **d.** the range of f $[-9,8]$

In Exercises 1–4, the graph below defines a function, f. Determine the following:



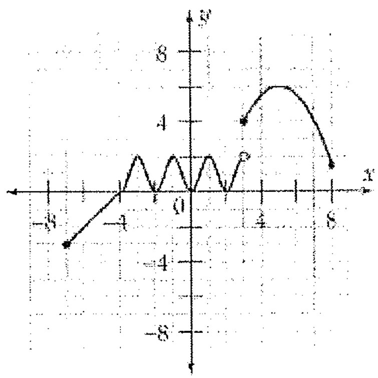
1. $f(-5)$

2. $f(1)$

3. the domain of f $[-6,9]$

4. the range of f $[0,9]$

In Exercises 5–8, the graph below defines a function, g. Determine the following:



5. $g(1)$

6. $g(5)$

7. the domain of g

8. the range of g

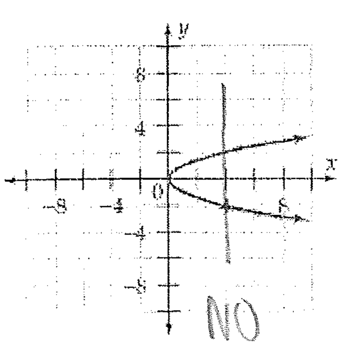
Vertical Line Test

A graph in a coordinate plane represents a function if and only if no vertical line intersects the graph more than once.

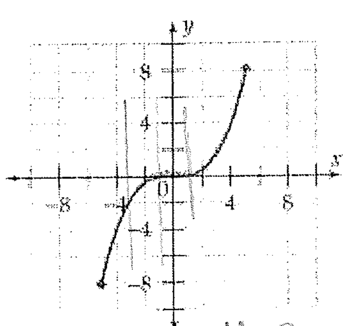
Example 2 Determining Whether a Graph Defines a Function

Use the Vertical Line Test to determine whether the following graphs represent functions. If not, give an example of an input value that corresponds to more than one output value.

To test whether or not a relation is a function given a graph, simply draw a vertical line through the graph. If it hits the graph in only one spot, it's a function!



NO

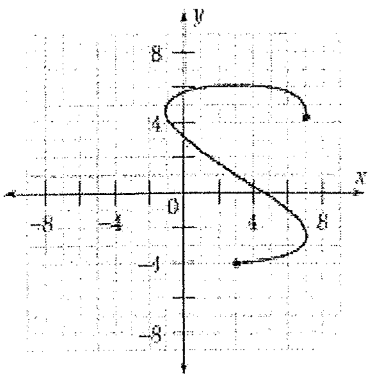


yes

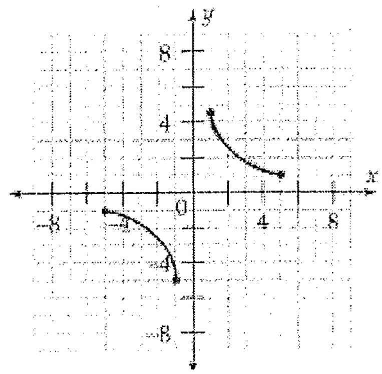
Figure 3.2-2

In Exercises 9–14, use the Vertical Line Test to determine whether the graph defines a function. If not, give an example of an input value that corresponds to more than one output value.

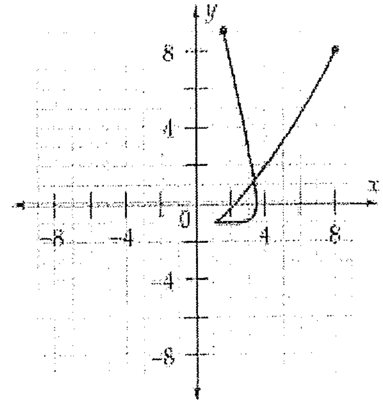
9.



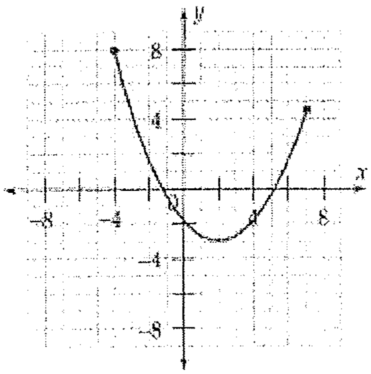
10.



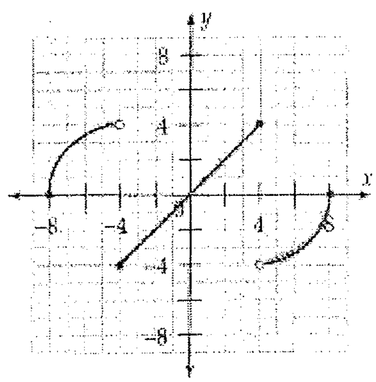
11.



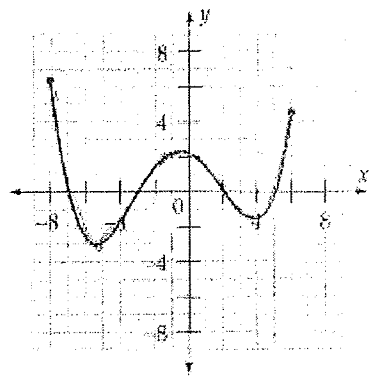
12.



13.



14.



Example 6 Graphing a Piecewise-Defined Function

Graph the piecewise-defined function below.

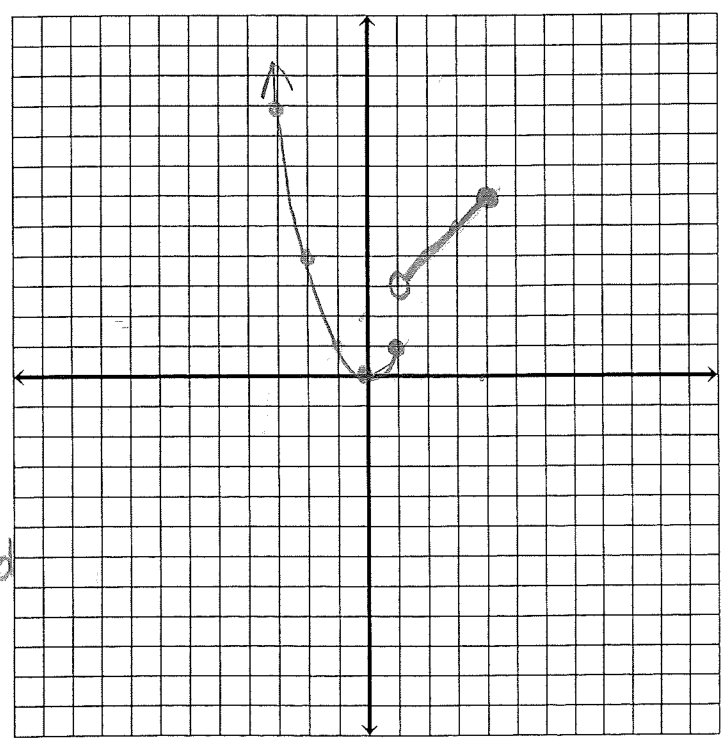
$$f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ x + 2 & \text{if } 1 < x \le 4 \end{cases}$$

$Y = x^2$

x	y
1	1
0	0
-1	1
-2	4
-3	9

$Y = x + 2$

	x	y	
*	1	3	open
	2	4	
	3	5	
	4	6	Closed



Technology Tip

Inequality symbols are in the **TEST** menu of TI-83/86, in the **TESTS** submenu of the HP-38 **MATH** menu, and in the **INEQ** submenu of the Sharp 9600 **MATH** menu. TI-89 has the symbols <, >, and | on the keyboard, and other inequality symbols and logical symbols (such as "and") are in the **TEST** submenu of the **MATH** menu.

Graphing Exploration

Graph the function f from Example 6 on a calculator as follows: On Sharp 9600 or HP-38 or TI-83/86 calculators, graph these two equations on the same screen:

$$Y_1 = X^2/(X \leq 1)$$
$$Y_2 = X + 2/((X > 1)(X \leq 4))$$

On a TI-89/92, graph these equations on the same screen:

$$Y_1 = X^2 \mid X \leq 1$$
$$Y_2 = X + 2 \mid X > 1 \quad \text{and} \quad X \leq 4$$

To graph f on a Casio 9850, graph these equations on the same screen (including commas and square brackets):

$$Y_1 = X^2,[-6, 1]$$
$$Y_2 = X + 2,[1, 4]$$

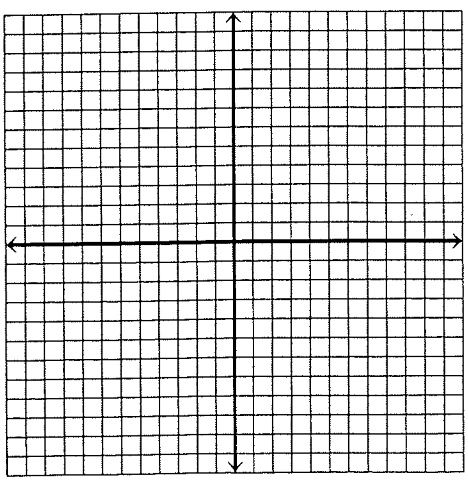
How does your graph compare with Figure 3.2-12?

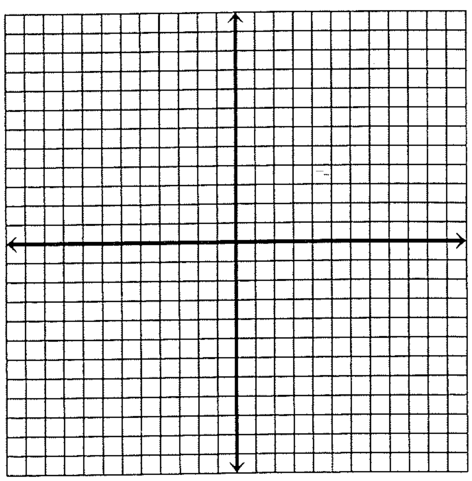
In Exercises 41–44, sketch the graph of the function. Be sure to indicate which endpoints are included and which are excluded.

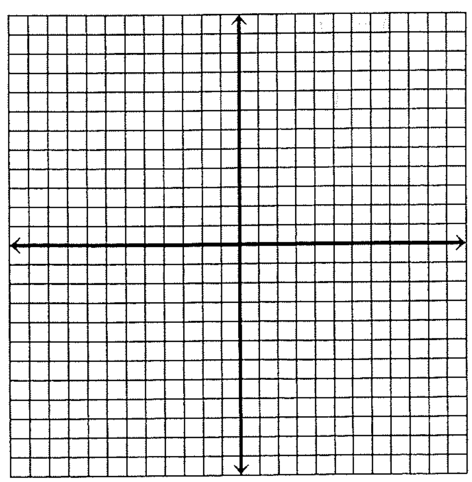
41. $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ x^2 & \text{if } x \geq -1 \end{cases}$

42. $g(x) = \begin{cases} |x| & \text{if } x < 1 \\ -3x + 4 & \text{if } x \geq 1 \end{cases}$

44. $f(x) = \begin{cases} x^2 & \text{if } x < -2 \\ x & \text{if } -2 \leq x < 4 \\ \sqrt{x} & \text{if } x \geq 4 \end{cases}$







Example 7 The Absolute-Value Function

Graph $f(x) = |x|$.

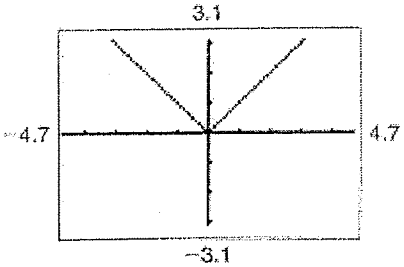


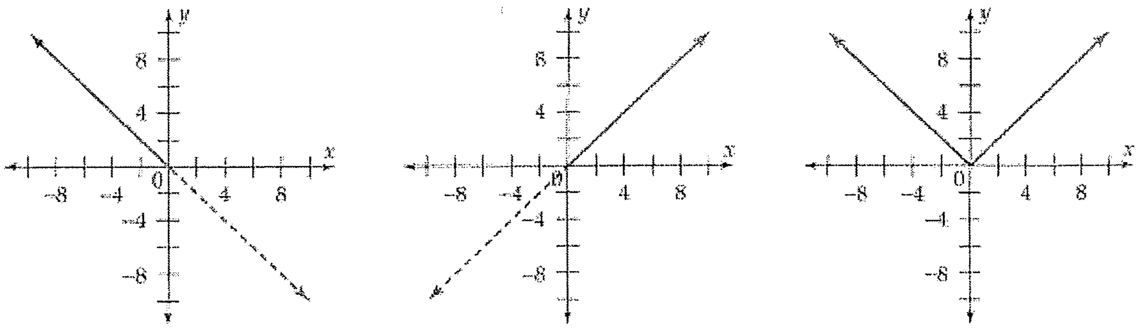
Figure 3.2-13

Solution

The absolute-value function $f(x) = |x|$ is also a piecewise-defined function, since by definition

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Its graph can be obtained by drawing the part of the line $y = x$ to the right of the origin and the part of the line $y = -x$ to the left of the origin or by graphing Y_1 = ABS X on a calculator.



In Exercises 45–49,

a. **Use the fact that the absolute-value function is piecewise-defined (see Example 7) to write the rule of the given function as a piecewise-defined function whose rule does not include any absolute value bars.**

b. **Graph the function.**

45. $f(x) = |x| + 2$

$$f(x) = \begin{cases} x + 2, & x \geq 0 \\ -x + 2, & x < 0 \end{cases}$$

46. $g(x) = |x| - 4$

$$f(x) = \begin{cases} x - 4, & x \geq 0 \\ -x - 4, & x < 0 \end{cases}$$

48. $g(x) = |x + 3|$

$$f(x) = \begin{cases} x + 3, & x \geq -3 \\ -x - 3, & x < -3 \end{cases}$$

49. $f(x) = |x - 5|$

$$f(x) = \begin{cases} x - 5, & x \geq 5 \\ -x + 5, & x < 5 \end{cases}$$