4-5 Quadratic Equations

1 Use the zero-factor property to solve equations.

2 Use factoring to solve equations.

3 Use factoring to solve applications.

Understanding Algebra

Remember that an *equation* must contain an *equals sign* (=). For a quadratic equation to be in standard form, $ax^2 + bx + c$ must be on one side of the equals sign and 0 must be on the other side.

Examples of Quadratic Equations

$$3x^2 + 6x - 4 = 0$$
$$5x = 2x^2 - 4$$
$$(x + 4)(x - 3) = 0$$

Any quadratic equation can be written in **standard form.**

Standard Form of a Quadratic Equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

where a, b, and c are real numbers.

Before going any further, make sure that you can rewrite each of the three quadratic equations given above in standard form, with $a > 0$.

1 Use the Zero-Factor Property to Solve Equations

To solve equations using factoring, we use the **zero-factor property**.

Zero-Factor Property

For all real numbers a and b, if $a \cdot b = 0$, then either $a = 0$ or $b = 0$, or both a and $b = 0$.

The zero-factor property states that *if the product of two factors equals 0, one (or both) of the factors must be 0.*

$x^2 + 2x - 15 = 0$

EXAMPLE 1 Solve the equation $(x + 5)(x - 3) = 0$.

$x = -5 \quad x = 3$

Solve.

7. $x(x+3)=0$

$x=0 \quad x=-3$

8. $x(x-2)=0$

$x=0 \quad x=2$

9. $4x(x-1)=0$

$x=0 \quad x=1$

10. $5x(x+6)=0$

$x=0 \quad x=-6$

11. $\cancel{2}(x+1)(x-7)=0$

$x=-1 \quad x=7$

12. $3(a-5)(a+2)=0$

$a=5 \quad a=-2$

13. $x(x-9)(x-4)=0$

$x=0,\ x=9,\ x=4$

14. $2a(a+3)(a+10)=0$

$a=0 \quad a=-3 \quad a=-10$

15. $(3x-2)(7x-1)=0$

$3a-2=0 \quad 7a-1=0$

$a=2/3 \quad a=1/7$

$(3a-4)(6a+7)=0$

$a=4/3 \quad a=-7/6$

2 Use Factoring to Solve Equations

Following is a procedure that can be used to obtain the solution to an equation by factoring.

To Solve an Equation by Factoring

1. Use the addition property to remove all terms from one side of the equation. This will result in one side of the equation being equal to 0.
2. Combine like terms in the equation and then factor.
3. Set each factor *containing a variable* equal to 0, solve the equations, and find the solutions.
4. Check the solutions in the *original* equation.

Helpful Hint

If you do not remember how to factor, review Sections 5.3–5.7.

EXAMPLE 2 Solve the equation $4x^2 = 24x$.

$4x^2-24x=0$

$4x(x-6)=0$

$x=0 \quad x=6$

2 Use Factoring to Solve Equations

Following is a procedure that can be used to obtain the solution to an equation by factoring.

To Solve an Equation by Factoring

1. Use the addition property to remove all terms from one side of the equation. This will result in one side of the equation being equal to 0.
2. Combine like terms in the equation and then factor.
3. Set each factor *containing a variable* equal to 0, solve the equations, and find the solutions.
4. Check the solutions in the *original* equation.

EXAMPLE 2 Solve the equation $4x^2 = 24x$.

17. $4x^2 = 12x$

$4x^2-12x=0$
$4x(x-3)=0$
$x=0 \quad x=3$

18. $3y^2 = -21y$

$3y^2+21y=0$
$3y(y+7)=0$
$y=0 \quad y=-7$

19. $x^2 + 5x = 0$

$x(x+5)=0$
$x=0 \quad x=-5$

20. $4a^2 - 32a = 0$

$4a(a-8)=0$
$a=0 \quad a=8$

21. $-x^2 + 6x = 0$

$-x(x-6)=0$
$x=0 \quad x=6$

25. $a^2 + 6a + 5 = 0$

$(a+1)(a+5)=0$
$a=-1 \quad a=-5$

26. $x^2 - 6x + 5 = 0$

$(x-5)(x-1)=0$
$x=5 \quad x=1$

27. $x^2 + x - 12 = 0$

$(x+4)(x-3)=0$
$x=-4 \quad x=3$

28. $b^2 + b - 72 = 0$

$(b+9)(b-8)=0$
$b=-9 \quad b=8$

29. $x^2 + 8x + 16 = 0$

$(x+4)(x+4)=0$
$x=-4$

30. $c^2 - 12c = -36$

$c^2-12c+36=0$
$(c-6)^2=0$
$c=6$

EXAMPLE 3 Solve the equation $(x-1)(3x+2) = 4x$.

$3x^2+2x-3x-2=4x$

$3x^2-1x-2=4x$

$3x^2-5x-2=0$

$3x^2-6x+1x-2=0$

$3x(x-2)+1(x-2)=0$

$(x-2)(3x+1)=0$

$ac=-6$

$x=2$

$x=-1/3$

31. $(2x+5)(x-1) = 12x$

$2x^2-2x+5x-5=12x$

$2x^2-9x-5=0$

$(2x+1)(x-5)=0$

$x=-1/2 \quad x=5$

32. $a(a+2) = 48$

$a^2+2a=48$

$a^2+2a-48=0$

$(a+8)(a-6)=0$

$a=-8 \quad a=6$

EXAMPLE 4 Solve the equation $3x^2 + 2x + 12 = -13x$

$3x^2+15x+12=0$

$3(x^2+5x+4)=0$

$3(x+4)(x+1)=0$

$x=-4 \quad x=-1$

55. $6a^2 - 12 - 4a = 19a - 32$

$+32 \; -19a \quad -19a \; +32$

$6a^2-23a+20=0$

$6a^2-8a-15a+20=0$

$2a(3a-4)-5(3a-4)=0$

$(3a-4)(2a-5)=0$

$a=4/3$

$a=5/2$

56. $4(a^2-3) = 6a + 4(a+3)$

$4a^2-12=6a+4a+12$

$4a^2-10a-24=0$

$2(2a^2-5a-12)=0$

$2(2a+3)(a-4)=0$

$a=-3/2 \quad a=4$

The zero-factor property can be extended to three or more factors, as illustrated in Example 5.

EXAMPLE 5 Solve the equation $2p^3 + 5p^2 - 3p = 0$.

$p(2p^2+5p-3)=0$

$p(2p-1)(p+3)=0$

$p=0 \quad p=1/2 \quad p=-3$

37. $x^3 - 3x^2 = 18x$

$x^3-3x^2-18x=0$

$x(x^2-3x-18)=0$

$x(x-6)(x+3)=0$

$x=0 \quad x=6, \; x=-3$

38. $x^3 = -19x^2 + 42x$

39. $4c^3 + 4c^2 - 48c = 0$