9.3 PreCalculus Notes
Other Identities

<u>Other Identities</u>
<u>Double Angle Identities</u>

$\sin(2x) = 2sinxcosx$

$\cos(2x) = cos^2x - sin^2x$

$\tan(2x) = \dfrac{2tanx}{1 - tan^2x}$

Where do these identities come from?

Sine Double Angle Identity
$\sin(2x) = 2sinxcosx$

How can we rewrite $2x$ so that we can use one of our other identities?

$\sin(2x) = \sin(x+x) \quad 2x = x + x$

$= \sin x \cos x + \sin x \cos x$

$= 2 \sin x \cos x$

Cosine Double Angle Identity
$\cos(2x) = cos^2x - sin^2x$

$\cos(2x) = \cos(x+x)$

$= \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

Tangent Double Angle Identity
$\tan(2x) = \dfrac{2tanx}{1 - tan^2x}$

$\tan(2x) = \tan(x+x)$

$= \dfrac{\tan x + \tan x}{1 - \tan x \tan x}$

$= \dfrac{2 \tan x}{1 - \tan^2 x}$

Let's take a closer look at the cosine double angle identity.

Can we do anything with the terms of this identity?

$$\cos(2x) = cos^2x - sin^2x$$

$$\cos(2x) = \underline{\cos^2 x} - \sin^2 x$$

$$= (1 - \sin^2 x) - \sin^2 x$$

$$= \boxed{1 - 2\sin^2 x}$$

$$\cos(2x) = \cos^2 x - \underline{\sin^2 x}$$

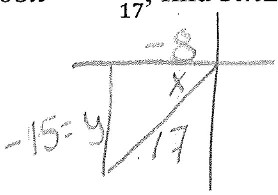
$$= \cos^2 x - (1 - \cos^2 x)$$

$$= \cos^2 x - 1 + \cos^2 x = \boxed{2\cos^2 x - 1}$$

Example 1:

If $\pi < x < \frac{3\pi}{2}$ and $cosx = -\frac{8}{17}$, find $sin2x, cos2x, and\ tan2x$.

3rd quad.



$$(-8)^2 + y^2 = 17^2$$

$$64 + y^2 = 289$$

$$y^2 = 225$$

$$y = 15$$

$$\sin 2x = 2\sin x\cos x$$

$$= 2\left(-\frac{15}{17} \cdot -\frac{8}{17}\right)$$

$$= 2\left(\frac{120}{289}\right)$$

$$= \frac{240}{286}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= \left(-\frac{8}{17}\right)^2 - \left(-\frac{15}{17}\right)^2$$

$$= \frac{64}{289} - \frac{225}{289}$$

$$= \frac{-161}{289}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$= \frac{2\left(\frac{-15}{-8}\right)}{1 - \left(\frac{-15}{-8}\right)^2}$$

$$= \frac{\frac{30}{8}}{1 - \frac{225}{64}} = \frac{\frac{30}{8}}{\frac{64}{64} - \frac{225}{64}}$$

$$= \frac{\frac{30}{8}}{\frac{-161}{64}} = \frac{30}{\cancel{8}} \cdot \frac{\cancel{64}^{\,8}}{-161}$$

$$= -\frac{240}{161}$$

Example 2:

Prove that $\frac{1-cos2x}{sin2x} = tanx$.

$$\frac{1-\cos 2x}{\sin 2x}$$

$$\frac{1-(\cos^2 x-\sin^2 x)}{2\sin x\cos x}$$

$$\frac{1-\cos^2 x+\sin^2 x}{2\sin x\cos x}$$

$$\frac{1-(1-\sin^2 x)+\sin^2 x}{2\sin x\cos x}$$

$$\frac{\cancel{1-1}+\sin^2 x+\sin^2 x}{2\sin x\cos x}$$

$$\frac{\cancel{2}\sin^2 x}{\cancel{2}\cancel{\sin x}\cos x}$$

$$\frac{\sin x}{\cos x}$$

$$\tan x$$

Example 3:

~~Prove that~~ $2sinxcos^3x - 2sin^3cosx$.

Simplify

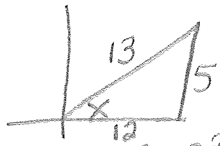
$$2\sin x\cos^3 x-2\sin^3 x\cos x$$

$$\underbrace{2\sin x\cos x}_{\sin(2x)}\,(\underbrace{\cos^2 x-\sin^2 x}_{\cos(2x)})$$

Exercises 9.3

In Exercises 23-30, find sin 2x, cos 2x, and tan 2x under the given conditions.

23. $sinx = \frac{5}{13}, for\ \underbrace{0 < x < \frac{\pi}{2}}_{}$ 1st quad



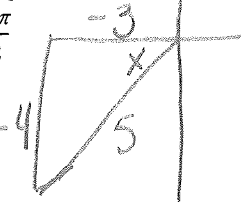
$\sin 2x = 2\sin x\cos x$
$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{120}{169}}$

$5^2 + x^2 = 13^2$
$25 + x^2 = 169$
$x^2 = 144$
$x = 12$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$
$= \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{\frac{120}{\cancel{169}}}{\frac{119}{\cancel{169}}} = \boxed{\frac{120}{119}}$

24. $sinx = -\frac{4}{5}, for\ \underbrace{\pi < x < \frac{3\pi}{2}}_{}$ 3rd quad



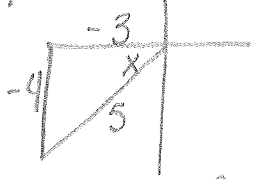
$\sin 2x = 2\sin x\cos x$
$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$
$= \boxed{\frac{24}{25}}$

$(-4)^2 + x^2 = 5^2$
$16 + x^2 = 25$
$x^2 = 9$
$x = -3$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$
$= \frac{9}{25} - \frac{16}{25} = \boxed{-\frac{7}{25}}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{\frac{24}{25}}{-\frac{7}{25}} = \boxed{-\frac{24}{7}}$

25. $cosx = -\frac{3}{5}, for\ \underbrace{\pi < x < \frac{3\pi}{2}}_{}$ 3rd quad



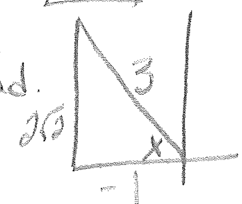
$\sin 2x = 2\sin x\cos x$
$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$
$= \frac{24}{25}$

$(-3)^2 + x^2 = 5^2$
$9 + x^2 = 25$
$x^2 = 16$
$x = -4$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$
$= \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{\frac{24}{25}}{-\frac{7}{25}} = \frac{24}{-7}$

26. $cosx = -\frac{1}{3}, for\ \underbrace{\frac{\pi}{2} < x < \pi}_{}$ 2nd quad.



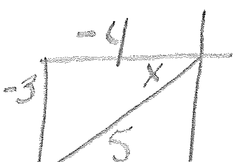
$\sin 2x = 2\sin x\cos x$
$= 2\left(\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{3}\right)$
$= -\frac{4\sqrt{2}}{9}$

$(-1)^2 + x^2 = 3^2$
$1 + x^2 = 9$
$x^2 = 8$
$x = \sqrt{8}$
$= 2\sqrt{2}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(-\frac{1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2$
$= \frac{1}{9} - \frac{8}{9} = -\frac{7}{9}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-\frac{4\sqrt{2}}{\cancel{9}}}{-\frac{7}{\cancel{9}}} = \frac{4\sqrt{2}}{7}$

27. $tanx = \frac{3}{4}, for\ \underbrace{\pi < x < \frac{3\pi}{2}}_{}$ 3rd quad.

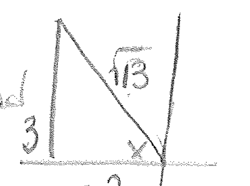


$\sin 2x = 2\sin x\cos x$
$= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$
$= \frac{24}{25}$

$(-3)^2 + (-4)^2 = x^2$
$9 + 16 = x^2$
$25 = x^2$
$5 = x$

$\cos 2x = \cos^2 x - \sin^2 x$

28. $tanx = -\frac{3}{2}, for\ \underbrace{\frac{\pi}{2} < x < \pi}_{}$ 2nd quad



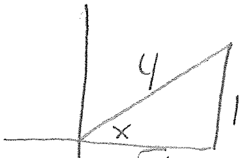
$\sin 2x = 2\sin x\cos x$
$= 2\left(\frac{3}{\sqrt{13}}\right)\left(\frac{-2}{\sqrt{13}}\right)$
$= -\frac{12}{13}$

$3^2 + (-2)^2 = x^2$
$9 + 4 = x^2$
$13 = x^2$
$\sqrt{13} = x$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(\frac{-2}{\sqrt{13}}\right)^2 - \left(\frac{3}{\sqrt{13}}\right)^2$
$= \frac{4}{13} - \frac{9}{13} = \frac{-5}{13}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} \longrightarrow = \frac{\frac{-12}{13}}{\frac{-5}{13}} = \frac{12}{5}$

29. $cscx = \frac{4}{1}, for\ 0 < x < \frac{\pi}{2}$



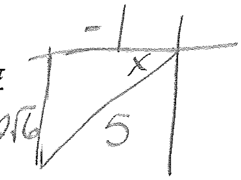
$1^2 + x^2 = 4^2$
$1 + x^2 = 16$
$x^2 = 15$
$x = \sqrt{15}$

$\sin 2x = 2\sin x \cos x$
$= 2\left(\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right)$
$= \frac{2\sqrt{15}}{16}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(\frac{\sqrt{15}}{4}\right)^2 - \left(\frac{1}{4}\right)^2$
$= \frac{15}{16} - \frac{1}{16} = \frac{14}{16} = \frac{7}{8}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{\frac{2\sqrt{15}}{16}}{\frac{14}{16}} = \frac{2\sqrt{15}}{14} = \frac{\sqrt{15}}{7}$

30. $secx = -5, for\ \pi < x < \frac{3\pi}{2}$



$(-1)^2 + x^2 = 5^2$
$1 + x^2 = 25$
$x^2 = 24$
$x = -2\sqrt{6}$

$\sin 2x = 2\left(-\frac{2\sqrt{6}}{5}\right)\left(\frac{-1}{5}\right)$
$= \frac{4\sqrt{6}}{25}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= \left(\frac{-1}{5}\right)^2 - \left(-\frac{2\sqrt{6}}{5}\right)^2$
$= \frac{1}{25} - \frac{24}{25} = \frac{-23}{25}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{\frac{4\sqrt{6}}{25}}{\frac{-23}{25}} = -\frac{4\sqrt{6}}{23}$

In Exercises 37-42, assume sin x = $\frac{0.6}{1}$ and 0 < x < $\frac{\pi}{2}$ and evaluate the given expression.

38. $cos4x$

$\cos 4x = 1 - 2\sin^2(2x)$
$= 1 - 2(2\sin x \cos x)^2$
$= 1 - 2(2(0.6)(0.8))^2$
$= -0.8432$

39. $cos2x$

$\cos 2x = 1 - 2\sin^2 x$
$= 1 - 2(0.6)^2$
$= 0.28$

40. $sin4x$

$\sin 4x = 2\sin 2x \cos 2x$
$= 2(2\sin x \cos x)(\cos^2 x - \sin^2 x)$
$= 2(2(0.6)(0.8))(.8^2 - .6^2)$
$= 0.5376$

In Exercises 45-50, simplify the given expression.

45. $\frac{sin2x}{2sinx}$

$= \frac{\cancel{2}\cancel{\sin x}\cos x}{\cancel{2}\cancel{\sin x}}$

$= \cos x$

47. $2cos2ysin2y$

$= \sin 2(2y)$
$= \sin 4y$

49. $(sinx + cosx)^2 - sin2x$

$= \sin^2 x + \cancel{2\sin x \cos x} + \cos^2 x$
$- \cancel{2\sin x \cos x}$
$= (\sin^2 x + \cos^2 x)$
$= 1$