2.2 Solving Quadratic Equations Algebraically

Definition of a Quadratic Equation

A *quadratic*, or *second degree*, equation is one that can be written in the form

$$ax^2 + bx + c = 0$$

for real constants a, b, and c, with $a \neq 0$.

NOTE This chapter considers only **real solutions**, that is, solutions that are real numbers.

Techniques Used to Solve Quadratic Equations

There are four techniques normally used to algebraically find exact solutions of quadratic equations. Techniques that can be used to solve *some* quadratic equations include

- factoring
- taking the square root of both sides of an equation

Techniques that can be used to solve *all* quadratic equations include

- completing the square
- using the quadratic formula

Solving Quadratic Equations by Factoring

The factoring method of solving quadratic equations is based on the Zero Product Property of real numbers.

The Zero Product Property

If a product of real numbers is zero, then at least one of the factors is zero. In other words,

If $ab = 0$, then $a = 0$ or $b = 0$ (or both).

Example 1 Solving a Quadratic Equation by Factoring

These are like 1-12.

Solve $3x^2 - x = 10$ by factoring.

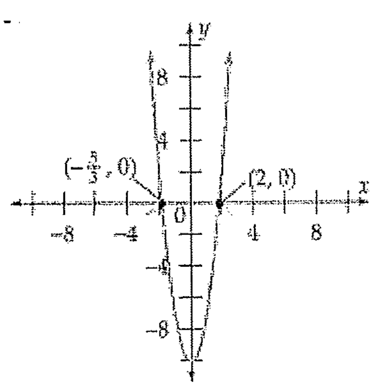


Figure 2.2-1

$$3x^2 - x = 10$$
$$3x^2 - x - 10 = 0$$
$$(3x + 5)(x - 2) = 0$$
$$x = -5/3 \qquad x = 2$$

These are like 13-24.

$4x^2 = 9$

Taking the Square Root of Both Sides of an Equation

> **CAUTION**
> When taking the square root of both sides of an equation, remember to write ± on one side of the equation.

Example 2 **Solving $ax^2 = b$**

Solve $\frac{3x^2}{3} = \frac{9}{3}$.

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

The method of taking the square root of both sides of an equation can be used to solve equations of the form $a(x-h)^2 = k$.

Example 3 **Solving $a(x-h)^2 = k$**

Solve $\frac{2(x+4)^2}{2} = \frac{6}{2}$.

$\sqrt{(x+4)^2} = \sqrt{3}$

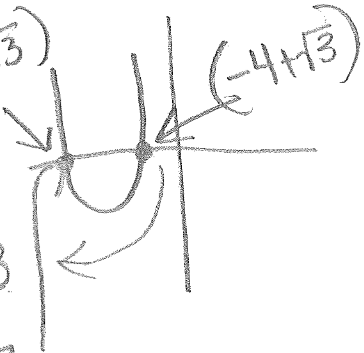
$x + 4 = \pm\sqrt{3}$

$x = -4 \pm \sqrt{3}$

↓ exact answer

$-4+\sqrt{3} \approx -2.3$

$-4-\sqrt{3} \approx -5.7$



Example 4 **Solving a Quadratic Equation by Completing the Square**

> Like 25-52

Solve $2x^2 - 6x + 1 = 0$ by completing the square.

$\frac{2x^2}{2} - \frac{8x}{2} - \frac{8}{2} = \frac{0}{2}$

$x^2 - 4x - 4 = 0$

$x^2 - 4x + 4 = 4 + 4$

$\sqrt{(x-2)^2} = \sqrt{8}$

$x - 2 = \pm\sqrt{8}$

$x - 2 = \pm 2\sqrt{2}$

$x = 2 \pm 2\sqrt{2}$

The Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Because the quadratic formula can be used to solve *any* quadratic equation, it should be memorized.

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Example 5 Solving a Quadratic Equation by Using the Quadratic Formula

Like 29-40

Solve $x^2 + 3 = -8x$ by using the quadratic formula.

$x^2 + 8x + 3 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{64 - 4(1)(3)}}{2(1)}$

$\frac{-8 \pm \sqrt{64 - 12}}{2} = \frac{-8 \pm \sqrt{52}}{2}$

$52 \to 2 \cdot 26 \to 2 \cdot 13$

$\frac{\cancel{-8}^{4} \pm \cancel{2}^{1}\sqrt{13}}{\cancel{2}}$

$-4 \pm \sqrt{13}$

The Discriminant

The expression $b^2 - 4ac$ in the quadratic formula, called the discriminant, can be used to determine the *number* of real solutions of the equation $ax^2 + bx + c = 0$.

Real Solutions of a Quadratic Equation

Discriminant Value	Number of Real Solutions of $ax^2 + bx + c = 0$
$b^2 - 4ac > 0$	2 distinct real solutions
$b^2 - 4ac = 0$	1 distinct real solution
$b^2 - 4ac < 0$	0 real solutions

The discriminant can be used to determine if an equation has no real solutions without completing all computations.

Example 6 Determining the Number of Solutions by Using the Discriminant

Like 41-46

Solve $2x^2 = -x - 3$.

$2x^2 + x + 3 = 0$

$b^2 - 4ac$

$1^2 - 4(2)(3)$

$1 - 24 = -23$

No Sol.

Polynomial Equations in Quadratic Form

Like 61-68

Example 7 Solving an Equation in Quadratic Form

Solve $4x^4 - 13x^2 + 3 = 0$.

$$(4x^2 - 1)(x^2 - 3) = 0$$

$$4x^2 - 1 = 0 \qquad x^2 - 3 = 0$$

$$\sqrt{x^2} = \sqrt{\tfrac{1}{4}} \qquad x^2 = 3$$

$$x = \pm \tfrac{1}{2} \qquad x = \pm\sqrt{3}$$

In Exercises 69–72, find a number k such that the given equation has exactly one real solution.

Like 71

$d = 0$

69. $x^2 + kx + 25 = 0$

$$b^2 - 4ac = 0$$

$$K^2 - 4(1)(25) = 0$$

$$K^2 - 100 = 0$$

$$\sqrt{K^2} = \sqrt{100}$$

$$K = \pm 10$$

PreCalc 2.2 Homework

Name_______________________________

In Exercises 1–12, solve each equation by factoring.

1. $x^2 - 8x + 15 = 0$

3. $x^2 - 5x = 14$

5. $2y^2 + 5y - 3 = 0$

7. $4t^2 + 9t + 2 = 0$

9. $3u^2 + u = 4$

11. $12x^2 + 13x = 4$

In Exercises 13–24, solve the equation by taking the square root of both sides. Give exact solutions and approximate solutions, if appropriate.

15. $x^2 = 40$

17. $3x^2 = 12$

19. $-5s^2 = -30$

21. $25x^2 - 4 = 0$

In Exercises 25–28, solve the equation by completing the square.

25. $x^2 - 2x = 12$

26. $x^2 - 4x - 30 = 0$

In Exercises 29–40, use the quadratic formula to solve the equation.

29. $x^2 - 4x + 1 = 0$

31. $x^2 + 6x + 7 = 0$

39. $5u^2 + 8u = -2$

In Exercises 41–46, find the number of real solutions of the equation by computing the discriminant.

41. $x^2 + 4x + 1 = 0$

43. $9x^2 = 12x + 1$

45. $25t^2 + 49 = 70t$

In Exercises 61–68, find all exact real solutions of the equation.

61. $y^4 - 7y^2 + 6 = 0$

63. $x^4 - 2x^2 - 35 = 0$

65. $2y^4 - 9y^2 + 4 = 0$

In Exercises 69–72, find a number k such that the given equation has exactly one real solution.

69. $x^2 + kx + 25 = 0$

71. $kx^2 + 8x + 1 = 0$