PreCalculus
"Can do Calculus" Notes and Examples

Name Key
Date ______ Hour ______

The terms of Geometric series grow very rapidly when the common ratio is greater than 1. On the other hand they decrease rapidly if the common ratio is less than 1. In fact they decrease so rapidly that an INFINITE SERIES has a FINITE Sum.

$* \frac{1}{2} = r$

For example, think about the series $16+8+4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\ldots$ ∞. The terms of the series are getting smaller and smaller, so the sum will approach a set number (in this case 32).

Sum of an Infinite Geometric Series

If a geometric sequence has a common ratio r and $|r|<1$, then the sum, S, of the related infinite series is as follows: $S=\frac{a_1}{1-r}$ $\quad S_\infty = \frac{a_1}{1-r}$

An infinite geometric series with $|r|\geq 1$ does **not** have a Sum

Alg 2
To say that an infinite series $a_1+a_2+a_3+\ldots$ has a sum means that the sequence of partial sums $S_1 = a_1$, $S_2 = a_1+a_2$, $S_3 = a_1+a_2+a_3$ $S_n = a_1+a_2+a_3+\ldots a_n$ to a number S as n gets very large

If a infinite series does not converge to a sum, the series must diverges. An infinite geometric series with $|r|\geq 1$ diverges and a sum can **not** be found.

To find the sum of an infinite series:

1) Determine if it is geometric. (Does it have a common ratio?)
2) Determine if it diverges or converges. (Is $|r|<1$)
   a. If it converges find the sum using $\boxed{S=\frac{a_1}{1-r}}$

   $r=-4$ b. If it diverges - no sum exists

converges = Can
diverges = don't

Example 1: Solve: $1+\frac{1}{2}+\frac{1}{4}+\ldots\ldots$ ∞ Find $r =$

Since $|r| = \frac{1}{2}$, the series converges → Can.

Find the sum

$$S_\infty = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{2}} \rightarrow \frac{1}{\frac{1}{2}} \leftarrow \text{flip N MULT.}$$

$$1 \cdot \frac{2}{1} = 2$$

∞ is the mathematical symbol for infinity.

infiNITY sum

The notation $\sum_{k=1}^{\infty}$ indicates an ______________________.

$$\frac{a_1}{1-r}$$

Example 2: **Solve:** Using Summation Notation

$$\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)\left(-\frac{8}{7}\right)^n$$

is this geo
$r = -8/7$

$n=0$ $\frac{3}{4}$ $n=1$ $\boxed{\frac{-24}{28}}$ $n=2$

$\frac{-24}{28} \div \frac{3}{4}$

$\frac{8}{7}\,\cancel{\frac{-24}{28}} \cdot \cancel{\frac{4}{3}}$

Since $|r| =$ 8/7 ← larger than 1, the series diverges

don't add!

Example 3: Find the sum of the infinite series if it exists *(notice the lower limit is k=0)*

$$\sum_{k=0}^{\infty}\left(\frac{3}{10^k}\right)$$

$n=0$ $n=1$ $n=2$

$3, \frac{3}{10}, \frac{3}{100}$

$\frac{3}{10} \div 3$

$\frac{3}{10} \cdot \frac{1}{3} = \frac{1}{10}$

$\frac{3}{100} \div \frac{3}{10}$

$\frac{3}{100} \cdot \frac{10}{3} = \frac{1}{10}$

$$S_\infty = \frac{a_1}{1-r}$$

$$\frac{3}{1-\frac{1}{10}} = \frac{3}{\frac{9}{10}} = \frac{10}{3}$$

geo.
$r = \frac{1}{10}$

Converges
"can"

100
$*\ .15$
15

Geometric Series Story Problem

If a series increases or ~~decreases~~ by a percent then the series is **geometric** and

decimal form

$r =$ 1 + r if increases

$r =$ 1 − r if decreases

\$ 20
$*\ 1.06$
21.20

20
$+\ 1.20$

.10

Example 4:
Robert would like to buy a new TV. He starts with \$25. Each month he plans to save 10% more than the previous month. How much money will he have saved at the end of 9 months?

$25, 27.5, 30.25$

$*1.10$

(1+ rate)

$$\frac{a_1(1-r^n)}{1-r}$$

$$\frac{25(1-1.10^9)}{1-1.10}$$

$= 339.49$

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