2.4 Page 116 1-69 every other odd

Skip 45

2.4 Other Types of Equations

Definition of Absolute Value

The **absolute value** of a number c is denoted $|c|$.
If c is a real number, the $|c|$ is the distance from c to 0.

Example 1 Using the Geometric Definition of Absolute Value

Solve $|x - 4| = 8$ using the geometric definition of absolute value.

$|x-4| = 8$

$x-4=-8$ or $x-4=8$

$+4 \quad +4 \qquad +4 \quad +4$

$x=-4 \qquad x=12$

Solution

The equation $|x - 4| = 8$ can be interpreted as

the distance from x to 4 is 8 units.

See Figure 2.4-4:

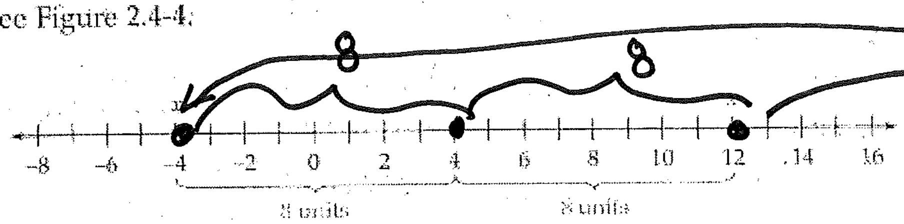

Figure 2.4-4

$|x-4| = -8$

No Sol.

The two possible values of x that are solutions of the original equation are -4 and 12, as shown.

Extraneous Solutions

As shown in Example 2 below, some solutions do not make the original equation true when checked by substitution. Such "fake" solutions are called **extraneous solutions,** or **extraneous roots.** Because extraneous solutions may occur when solving absolute-value equations, all solutions *must* be checked by substituting into the original equation or by graphing.

Example 2 Using the Algebraic Definition of Absolute Value

Solve $|x + 4| = 5x - 2$ by using the algebraic definition of absolute value.

$x+4=-(5x-2)$
$x+4= -5x+2$
$6x = -2 \quad \boxed{\cancel{x=-1/3}}$

$x+4=5x-2$
$6=4x$
$\frac{3}{2}=x$

Example 3 Solving an Absolute Value Equation

Solve $|x^2 + 4x - 3| = 2$.

$x^2+4x-3=-2$
$x^2+4x-1=0$

$x^2+4x-3=2$
$\quad -2 \ -2$
$x^2+4x-5=0$
$(x+5)(x-1)=0$

$x=-5$
$x=1$

$x^2+4x-1=0$
$x^2+4x+4=1+4$
$\sqrt{(x+2)^2}=\sqrt{5}$
$x+2=\pm\sqrt{5}$
$x=-2\pm\sqrt{5}$

Example 4 Solving a Radical Equation

Solve $5 + \sqrt{3x - 11} = x$

$(\sqrt{3x-11})^2 = (x-5)^2$

$3x - 11 = (x-5)(x-5)$

$3x - 11 = x^2 - 5x - 5x + 25$

$\cancel{3x} - 11 = x^2 - 10x + 25$
$-3x \qquad\qquad -3x$

$-11 = x^2 - 13x + 25$
$+11 \qquad\qquad +11$

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$

$x = 9 \qquad \cancel{x = 4}$ extraneous

Example 5 Using the Power Principle Twice

Solve $\sqrt{2x - 3} - \sqrt{x + 7} = 2$.

$(\sqrt{2x-3})^2 = (2 + \sqrt{x+7})^2$

$2x - 3 = (2 + \sqrt{x+7})(2 + \sqrt{x+7})$

$2x - 3 = \cancel{4} + 4\sqrt{x+7} + x + \cancel{7}$
$-11 \qquad -4 \qquad\qquad -7$

$2x - 14 = 4\sqrt{x+7} + \cancel{x}$
$-x \qquad\qquad -x$

$(x-14)^2 = (4\sqrt{x+7})^2$

$(x-14)(x-14) = 16(x+7)$

$x^2 - 28x + 196 = \cancel{16x} + 112$
$-16x - 112 \qquad -16x - 112$

$x^2 - 44x + 84 = 0$

$(x-2)(x-42) = 0$

$x = 2 \qquad x = 42$

extraneous

Example 7 Solving a Fractional Equation

Solve $\dfrac{6x^2 - x - 1}{2x^2 + 9x - 5} = 0$. ← set num = 0

$\dfrac{0}{5} = 0 \qquad \dfrac{5}{0} = \text{und.}$

$6x^2 - x - 1 = 0$

$(3x + 1)(2x - 1) = 0$

$x = -1/3 \qquad \boxed{\cancel{x = 1/2}}$

$2x^2 + 9x - 5$

$(2x - 1)(x + 5) = 0$

$x = 1/2 \qquad x = -5$

$$\cfrac{\cfrac{(3x+1)\cancel{(2x-1)}}{\cancel{(2x-1)}(x+5)}}{\cfrac{(x-3)\cancel{(x-2)}}{(x^2-4)}} = 0$$

$\cancel{(x-2)}(x+2)$

In Exercises 64–67, assume that all letters represent positive numbers. Solve each equation for the required letter.

64. $T = 2\pi\sqrt{\dfrac{m}{g}}$ for g

$\left(\dfrac{T}{2\pi}\right)^2 = \left(\sqrt{\dfrac{m}{g}}\right)^2$

$\dfrac{T^2}{4\pi^2} = \dfrac{m}{g}$

$m \cdot 4\pi^2 = T^2 g$

$\dfrac{4\pi^2 m}{T^2} = g$