Name:________________________ Date:______________ Hour:________

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2.5 Inequalities

Interval Notation

Let c and d be real numbers with $c < d$.

$[c, d]$ denotes the set of all real numbers x such that $c \le x \le d$.

(c, d) denotes the set of all real numbers x such that $c < x < d$.

$[c, d)$ denotes the set of all real numbers x such that $c \le x < d$.

$(c, d]$ denotes the set of all real numbers x such that $c < x \le d$.

REMINDER: (use parentheses) when you see < and >
[use brackets] when you see ≤ and ≥

Examples:

Represent the given intervals on a number line

a. $(3,9]$

3 9

b. $[-3,6]$

-3 6

Use interval notation to denote the set of all real numbers x that satisfy the given inequality. Draw a graph.

a. $-4 < x \le 5$

$(-4,5]$

b. $x \le -4$

$(-\infty,-4]$

Basic Principles for Solving Inequalities

Performing any of the following operations on an inequality produces an equivalent inequality.

1. Add or subtract the same quantity on both sides of the inequality.
2. Multiply or divide both sides of the inequality by the same *positive* quantity.
3. Multiply or divide both sides of the inequality by the same *negative* quantity, and *reverse the direction of the inequality.*

Examples: Solve the inequalities.

a. $5-3x > 7x-3$

$+3x \quad +3x$

$5 > 10x - 3$

$+3 \qquad +3$

$\frac{8}{10} > \frac{10x}{10}$

$\frac{4}{5} > x$

$x < \frac{4}{5}$

4/5

$(-\infty, \frac{4}{5})$

b. $\frac{(x-1)}{4} + 2x > \frac{2x-1}{3} + 2 \cdot 12$

$3x - 3 + 24x > 8x - 4 + 24$

$27x - 3 > 8x + 20$

$19x > 23$

$x > \frac{23}{19}$

$\frac{23}{19}$

$(\frac{23}{19}, \infty)$

c. $2x + 7 \le -21$ or $3(x+2) > 10$

$-7 \quad -7$

$\frac{2x}{2} \le \frac{-28}{2}$

$x \le -14$ or

$3x + 6 > 10$

$\frac{3x}{3} > \frac{4}{3}$

$x > \frac{4}{3}$

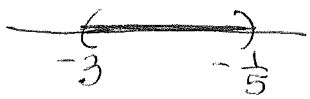
$-14 \qquad \frac{4}{3}$

$(-\infty, -14] \cup (\frac{4}{3}, \infty)$

c. $4 < 3 - 5x < 18$
$\quad -3 \quad -3 \qquad -3$

$\frac{1}{-5} < \frac{-5x}{-5} < \frac{15}{-5}$

$-\frac{1}{5} > x > -3$



$(-3, -\frac{1}{5})$

d. $2 \le 3x + 5 < 2x + 11$

$2 \le 3x + 5$ and $3x + 5 < 2x + 11$

$-5 \qquad -5 \qquad\qquad -2x \qquad -2x$

$\frac{-3}{3} \le \frac{3x}{3} \qquad\qquad 1x + 5 < 11$

$-1 \le x \qquad\qquad\qquad -5 \quad -5$

$x \ge -1$ and $1x < 6$

$[-1, 6)$

Example 4 Solving a Quadratic Inequality

Solve $2x^2 + 3x - 4 \le 0$.

$2x^2 + 3x - 4 = 0$

$(\quad)(\quad)$

Solution

The solutions of $2x^2 + 3x - 4 \le 0$ are the numbers x where the graph of $f(x) = 2x^2 + 3x - 4$ lies on or below the x-axis. The zeros of f can be found by using the quadratic formula.

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)} = \boxed{\frac{-3 \pm \sqrt{41}}{4}} \leftarrow \text{exact}$$

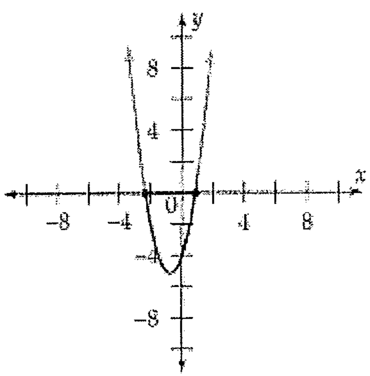


Figure 2.5-5

As shown in Figure 2.5-5, the graph lies below the x-axis *between* the two zeros. Therefore, the solutions of the original inequality are all numbers x such that

$$\frac{-3 - \sqrt{41}}{4} \le x \le \frac{-3 + \sqrt{41}}{4} \qquad \textit{Exact solution}$$

$$-2.35 \le x \le 0.85 \qquad \textit{Approximate solution}$$

$[-2.35, .85]$

Examples:

Solve the inequality. Write the answer in interval notation.

a. $(x+5)(x-2)^6(x-8) \le 0$

$\downarrow \quad \downarrow \quad \downarrow$

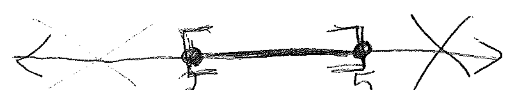
$-5 \quad 2 \quad 8$

$(-\infty, -5)$	-10	No
$(-5, 2)$	0	✓
$(2, 8)$		✓
$(8, \infty)$		No

b. $x^2 - 7x + 10 \le 0$

$(x-5)(x-2) \le 0$

$x = 5 \qquad x = 2$



$(-\infty, 2)$	0	False
$(2, 5)$	3	True
$(5, \infty)$	10	False

$[2, 5]$