6-5 Solving Square Root and Other Radical Equations

Content Standards

A.REI.2 Solve simple rational and radical equations in one variable, and . . . show how extraneous solutions may arise.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Objective To solve square root and other radical equations

$(\sqrt[2]{x})^2 = (4)^2$

$x^{1/2} = 4$

$(\sqrt[3]{x})^3 = (4)^3$

A **radical equation** is an equation that has a variable in a radicand or a variable with a rational exponent. If the radical has index 2, the equation is a **square root equation**. In this lesson, assume that all radicals and expressions with rational exponents represent real numbers.

Essential Understanding Solving a square root equation may require that you square each side of the equation. This can introduce extraneous solutions.

To solve a radical equation, isolate the radical on one side of the equation. Then raise each side to the power suggested by the index.

Problem 1 Solving a Square Root Equation

What is the solution of $\cancel{3} + \boxed{\sqrt{2x-3}} = 8$?

$-\cancel{3} \quad -3$

$(\sqrt{2x-3})^2 = (5)^2$

$2x - \cancel{3} = 25$

$+\cancel{3} \quad +3$

$\frac{2x}{2} = \frac{28}{2}$

$\boxed{x = 14}$

$3 + \sqrt{2 \cdot 14 - 3} \stackrel{?}{=} 8$

$3 + \sqrt{28 - 3} =$

$3 + \sqrt{25} =$

$3 + 5 =$

$8 \stackrel{\checkmark}{=} 8$

Got It? 1. What is the solution of $\sqrt{4x+1} - \cancel{5} = 0$?

$+\cancel{5} \quad +5$

$(\sqrt{4x+1})^2 = (5)^2$

$4x + \cancel{1} = 25$

$-\cancel{1} \quad -1$

$\frac{4x}{4} = \frac{24}{4}$

$\boxed{x = 6}$

Solve.

See Problem 1.

9. $3\sqrt{x} + \cancel{3} = 15$

$-\cancel{3} \quad -3$

$\frac{3\sqrt{x}}{3} = \frac{12}{3}$

$(\sqrt{x})^2 = (4)^2$

$x = 16$

10. $4\sqrt{x} - 1 = 3$

$\frac{4\sqrt{x}}{4} = \frac{4}{4}$

$(\sqrt{x})^2 = (1)^2$

$x = 1$

11. $(\sqrt{x+3})^2 = (5)^2$

$x + 3 = 25$

$-3 \quad -3$

$x = 22$

15. $(\sqrt{3x+4})^2 = (4)^2$

$3x + \cancel{4} = 16$

$-\cancel{4} \quad -4$

$3x = 12$

$x = 4$

16. $\sqrt{2x+3} - 7 = 0$

$(\sqrt{2x+3})^2 = (7)^2$

$2x + \cancel{3} = 49$

$-\cancel{3} \quad -3$

$2x = 46$

$x = 23$

17. $\sqrt{6-3x} - 2 = 0$

$(\sqrt{6-3x})^2 = (2)^2$

$\cancel{6} - 3x = 4$

$-\cancel{6} \quad -6$

$\frac{-3x}{-3} = \frac{-2}{-3}$ $\quad x = \frac{2}{3}$

To solve equations of the form $x^{\frac{m}{n}} = k$, raise each side of the equation to the power $\frac{n}{m}$, the reciprocal of $\frac{m}{n}$. If either m or n is even, then $\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = |x|$.

Problem 2 Solving Other Radical Equations

A What is the solution of $3(x+1)^{\frac{2}{3}} = 12$?

$$\frac{\cancel{3}\sqrt[3]{(x+1)^2}}{\cancel{3}} = \frac{12}{3} \quad \Longleftrightarrow \quad \frac{\cancel{3}(x+1)^{\frac{2}{3}}}{\cancel{3}} = \frac{12}{3}$$

$$\left(\sqrt[3]{(x+1)^2}\right)^3 = 4^3 \qquad (x+1)^{\frac{2}{3}\cdot 3} = 4^3$$

$$\sqrt{(x+1)^2} = \sqrt{64} \qquad \sqrt{(x+1)^2} = \sqrt{64}$$

$$x+1 = \pm 8 \qquad x+1 = \pm 8$$

$$\begin{aligned} -1 \\ x = \begin{cases} -1+8 = 7 \\ -1-8 = -9 \end{cases} \end{aligned}$$

B What is the solution of $3\sqrt[5]{(x+1)^3} + \cancel{1} = 25$?

$$-1 \quad -1$$

$$\frac{\cancel{3}\sqrt[5]{(x+1)^3}}{\cancel{3}} = \frac{24}{3}$$

$$\left(\sqrt[5]{(x+1)^3}\right)^5 = (8)^5$$

$$\sqrt[3]{(x+1)^3} = \sqrt[3]{32{,}768}$$

$$x+1 = 32$$

$$x = 31$$

Got It? 2. What are the solution(s) of $2(x+3)^{\frac{2}{3}} = 8$?

$$\frac{\cancel{2}(x+3)^{\frac{2}{3}}}{\cancel{2}} = \frac{8}{2}$$

$$(x+3)^{\frac{2}{3}} = 4$$

$$\left(\sqrt[3]{(x+3)^2}\right)^3 = (4)^3$$

$$\sqrt{(x+3)^2} = \sqrt{64}$$

$$x+\cancel{3} = \pm 8$$

$$-\cancel{3} \quad -3$$

Solve. See Problem 2.

18. $(x+5)^{\frac{2}{3}} = 4$

19. $(x+2)^{\frac{2}{3}} = 9$

20. $3(x-2)^{\frac{3}{4}} = 24$