

Applications of Equations

Objectives

- Solve application problems

Real-life situations are usually described verbally, but they must be interpreted and expressed as equivalent mathematical statements. The following guideline may be helpful.

Applied Problems Guideline

1. Read the problem carefully, and determine what is asked for.
2. Label the unknown quantities with variables.
3. Draw a picture of the situation, if appropriate.
4. Translate the verbal statements in the problem and the relationships between the known and unknown quantities into mathematical language.
5. Consolidate the mathematical information into an equation in one variable that can be solved or an equation in two variables that can be graphed.
6. Solve for at least one of the unknown quantities.
7. Find all remaining unknown quantities by using the relationships given in the problem.
8. Check and interpret all quantities found in the original problem.

Example 1

Number Relations

x y

The average of two real numbers is 41.125, and their product is 1683. Find the two numbers.

$$\frac{x+y}{2} = 41.125$$

$$x+y = 82.25$$

$$y = (82.25 - x)$$

$$xy = 1683$$

$$x(82.25 - x) = 1683$$

$$82.25x - x^2 = 1683$$

38.3 and 44

Example 2

Dimensions of a Rectangle

$$A = l \cdot w$$

A rectangle is twice as wide as it is high. If it has an area of 24.5 square inches, what are its dimensions?



$$\left\{ \begin{array}{l} w = 2l \\ 24.5 = l \cdot w \end{array} \right\}$$

$$24.5 = l \cdot 2l$$

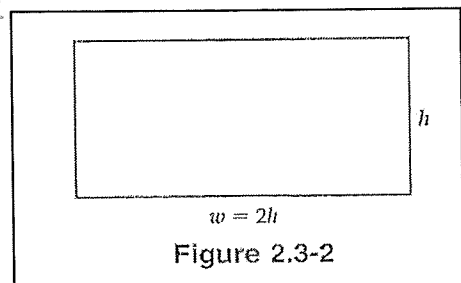
$$24.5 = \frac{2l^2}{2}$$

$$\sqrt{12.25} = \sqrt{l^2}$$

$$l = \pm 3.5$$

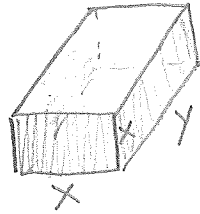
$$l = 3.5 \text{ inches}$$

$$w = 2(3.5) = 7 \text{ inches}$$



Example 3 Volume of a Rectangular Box

A rectangular box with a square base and no top is to have a volume of $30,000 \text{ cm}^3$. If the surface area of the box is 6000 cm^2 , what are the dimensions of the box?



$$V = l \cdot w \cdot h$$

$$V = x \cdot x \cdot y$$

$$30,000 = x^2 y$$

$$\left(\frac{30,000}{x^2}\right) = y$$

$$x^2 + 4xy = 6000$$

$$x^2 + 4x \left(\frac{30,000}{x^2}\right) = 6000$$

$$x^2 + \frac{120,000}{x} - 6000 = 0$$

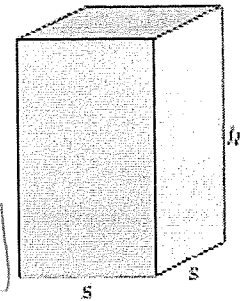


Figure 2.3-3

Example 4 Stock and Savings Returns

A high-risk stock pays dividends at a rate of 12% per year, and a savings account pays 6% interest per year. How much of a $\$9000$ investment should be put in the stock and how much should be put in savings to obtain a return of 8% per year on the total investment?

$$x + y = 9000$$

$$.12x + .06y = 720$$

$$-6(x + y = 9000) \quad (.08)(9000)$$

$$12x + 6y = 72000$$

$$\frac{12x + 6y = 72000}{-6x - 6y = -54000}$$

$$12x + 6y = 72,000$$

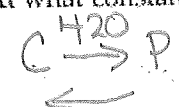
$$6x = 18,000$$

21.7 cm
64.29 cm

$x = 3000$ in 12%
 $y = 6000$ in 6%

Example 5 Distance

A pilot wants to make an 840 -mile round trip from Cleveland to Peoria and back in 5 hours flying time. There will be a headwind of 30 mph going to Peoria, and it is estimated that there will be a 40 -mph tailwind returning to Cleveland. At what constant engine speed should the plane be flown?



$$\frac{420}{r-30} + \frac{420}{r+40} = 5$$

$$D = \text{rate} \cdot \text{time}$$

$$420 = (r-30)t_1$$

$$\left(\frac{420}{r-30}\right) = t_1$$

$(r-w)$ = headwind
 $(r+w)$ = tailwind

$$D = \text{rate} \cdot \text{time}$$

$$420 = (r+40)t_2$$

$$\left(\frac{420}{r+40}\right) = t_2$$

$$t_1 + t_2 = 5$$

Example 6 Width of a Garden Walk

A landscaper wants to put a cement walk of uniform width around a rectangular garden that measures 24 by 40 feet. She has enough cement to cover 660 square feet. How wide should the walk be in order to use all the cement?

$$(24+2x)(40+2x) - (24 \cdot 40) = 660$$

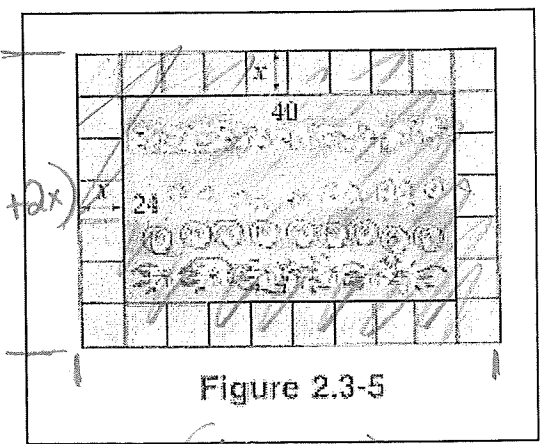
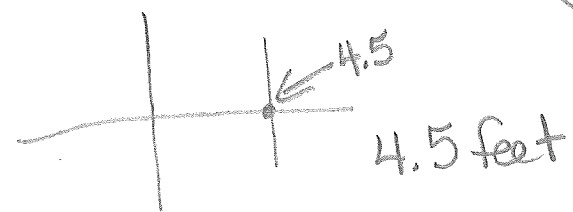


Figure 2.3-5

$$(40 + 2x)$$

Example 7 Box Construction

A box with no top that has a volume of 1000 cubic inches is to be constructed from a 22×30 -inch sheet of cardboard by cutting squares of equal size from each corner and folding up the flaps, as shown in Figure 2.3-6. What size square should be cut from each corner?

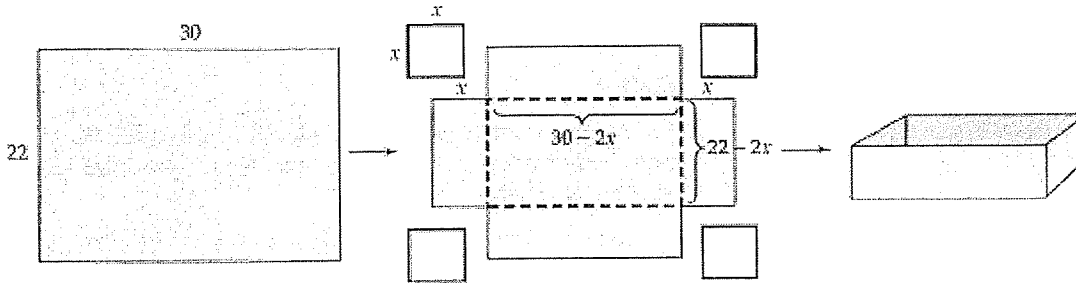


Figure 2.3-6

Length \times Width \times Height = Volume of the box

$$(30 - 2x)(22 - 2x)x = 1000$$

Example 8 Mixture Problem

A car radiator contains 12 quarts of fluid, 20% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the resulting mixture is 50% antifreeze?

2.3 Homework PreCalc

1. The sum of two numbers is 15 and the difference of their squares is 5. What are the numbers?
2. The sum of the squares of two consecutive integers is 4513. What are the integers?
3. A rectangle has a perimeter of 45 centimeters and an area of 112.5 square centimeters. What are its dimensions?
4. A triangle has an area of 96 square inches, and its height is two-thirds of its base. What are the base and height of the triangle?
8. A corner lot has dimensions 25 by 40 yards. The city plans to take a strip of uniform width along the two sides bordering the streets in order to widen these roads. How wide should the strip be if the remainder of the lot is to have an area of 844 square yards?
11. A radiator contains 8 quarts of fluid, 40% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 60% antifreeze?
12. A radiator contains 10 quarts of fluid, 30% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 40% antifreeze?
13. Two cars leave a gas station at the same time, one traveling north and the other south. The northbound car travels at 50 mph. After 3 hours the cars are 345 miles apart. How fast is the southbound car traveling?

14. An airplane flew with the wind for 2.5 hours and returned the same distance against the wind in 3.5 hours. If the cruising speed of the plane was a constant 360 mph in air, how fast was the wind blowing? *Hint:* If the wind speed is r miles per hour, then the plane travels at $(360 + r)$ mph with the wind and at $(360 - r)$ mph against the wind.

15. The average of two real numbers is 41.375 and their product is 1668. What are the numbers?

16. A rectangle is four times as long as it is wide. If it has an area of 36 square inches, what are its dimensions?

17. A 13-foot-long ladder leans on a wall. The bottom of the ladder is 5 feet from the wall. If the bottom is pulled out 3 feet farther from the wall, how far does the top of the ladder move down the wall? *Hint:* The ladder, ground, and wall form a right triangle. Draw pictures of this triangle before and after the ladder is moved. Use the Pythagorean Theorem to set up an equation.

19. Red Riding Hood drives the 432 miles to Grandmother's house in 1 hour less than it takes the Wolf to drive the same route. Her average speed is 6 mph faster than the Wolf's average speed. How fast does each drive?

20. To get to work Sam jogs 3 kilometers to the train, then rides the remaining 5 kilometers. If the train goes 40 km per hour faster than Sam's constant rate of jogging and the entire trip takes 30 minutes, how fast does Sam jog?

In Exercises 21–24, an object is thrown upward, dropped, or thrown downward and travels in a vertical line subject only to gravity with wind resistance ignored. The height h , in feet, of the object above the ground after t seconds is given by

$$h = -16t^2 + v_0t + h_0$$

where h_0 is the initial height of the object at starting time $t = 0$, and v_0 is the initial velocity (speed) of the object at time $t = 0$. The value of v_0 is taken as positive if the object starts moving upward at time $t = 0$ and negative if the object starts moving downward at $t = 0$. An object that is dropped (rather than thrown downward) has initial velocity $v_0 = 0$.

21. How long does it take an object to reach the ground in each case?
- It is dropped from the top of a 640-foot-high building.

b. It is thrown downward from the top of the same building, with an initial velocity of 52 feet per second.

22. You are standing on a cliff 200 feet high. How long will it take a rock to reach the ground at the bottom of the cliff in each case?
- You drop it.
 - You throw it downward at an initial velocity of 40 feet per second.
 - How far does the rock fall in 2 seconds if you throw it downward with an initial velocity of 40 feet per second.

23. A rocket is fired straight up from ground level with an initial velocity of 800 feet per second.
- How long does it take the rocket to rise 3200 feet?
 - When will the rocket hit the ground?

24. A rocket loaded with fireworks is to be shot vertically upward from ground level with an initial velocity of 200 feet per second. When the rocket reaches a height of 400 feet on its upward trip, the fireworks will be detonated. How many seconds after lift-off will this take place?

25. The dimensions of a rectangular box are consecutive integers. If the box has volume of 13,800 cubic centimeters, what are its dimensions?

26. Find a real number that exceeds its cube by 2.