2.1 Solving Equations Graphically

Key

Objectives

- Solve equations using the intersect method
- Solve equations using the x-intercept method

A **solution** of an equation is a number that, when substituted for the variable, produces a true statement. For example, 5 is a solution of $3x + 2 = 17$ because $3(5) + 2 = 17$ is a true statement. To **solve** an equation means to find all of its solutions.

Two equations are said to be **equivalent** if they have the same solutions. For example, $3x + 2 = 17$ and $x - 2 = 3$ are equivalent because 5 is the only solution of each equation.

The Intersection Method

To solve an equation of the form $f(x) = g(x)$ by using the *intersection method*, follow two steps.

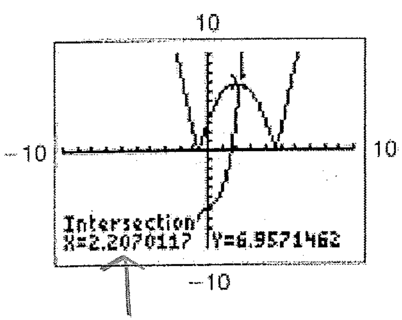
1. **Graph $y_1 = f(x)$ and $y_2 = g(x)$ on the same screen.**
2. **Find the x-coordinate of each point of intersection.**

Example 1 Solving an Equation Using the Intersect Method

Solve $|x^2 - 4x - 3| = x^3 + x - 6$.

Solution

Set $y_1 = |x^2 - 4x - 3|$ and $y_2 = x^3 + x - 6$.



To find the solution (intersection points), hit y= and graph the left side and right side separately.
Type an equation on each line separately
Hit graph
Hit 2nd calc(trace)
Intersection (5) enter
Arrow over to the left of the intersection
Hit enter, enter, enter
The answer will be displayed at the bottom
X=__________ y=___________

The x-Intercept Method

Follow three steps to solve an equation by the x-intercept method.

1. **Write the equation in the equivalent form $f(x) = 0$.**
2. **Graph $y = f(x)$.**
3. **Find the x-intercepts of the graph. The x-intercepts of the graph are the real solutions of the equation.**

Example 2 Solving an Equation by Using the x-Intercept Method

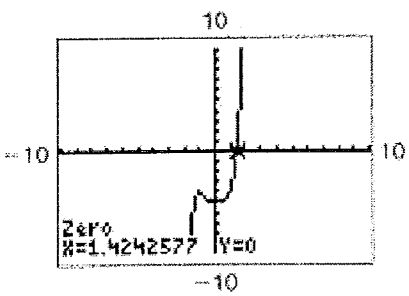
set $y_1 = x^5 + x^2$
$y_2 = x^3 + 5$

Solve the equation $x^5 + x^2 = x^3 + 5$.

Solution $Y_1 =$

Rewrite the equation so that one side is zero.

$$x^5 - x^3 + x^2 - 5 = 0$$



To find the solution (zeros or x intercepts), simply set the entire equation =0.
Hit y=
Type the equation
Hit graph
Hit 2nd calc(trace)
Zero
Left bound (estimate a number) enter
Right bound (estimate a number) enter
Guess (you can skip this) enter
The answer will be displayed at the bottom
X= 1.424 y= 0

Technological Quirks

Example 3 Solving $\sqrt{f(x)} = 0$ by Solving $f(x) = 0$

Solve $\sqrt{x^4 + x^2 - 2x - 1} = 0$. set only $x^4 + x^2 - 2x - 1$

Solution

Graph $y = \sqrt{x^4 + x^2 - 2x - 1}$. The trace feature may display no y-value for some points and the graphical zero finder may display an error message. See Figure 2.1-3 on the next page.

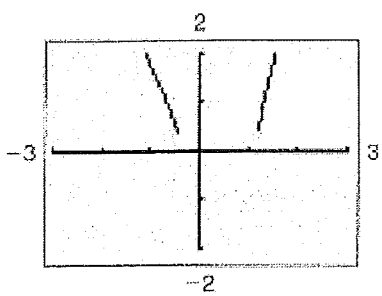


Figure 2.1-3

This difficulty can be eliminated by using the fact that the only number whose square root is zero is zero itself.

That is, the solutions of $\sqrt{x^4 + x^2 - 2x - 1} = 0$ are the same as the solutions of $x^4 + x^2 - 2x - 1 = 0$.

As the graphs below display, the solutions of $x^4 + x^2 - 2x - 1 = 0$ are

$$x \approx -0.4046978 \qquad \text{and} \qquad x \approx 1.1841347,$$

which are also approximate solutions of $\sqrt{x^4 + x^2 - 2x - 1} = 0$.

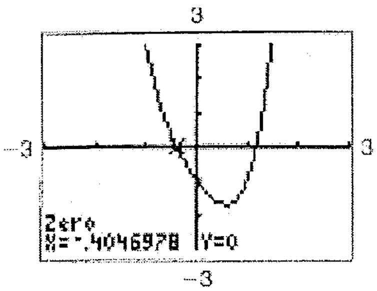


Figure 2.1-4a

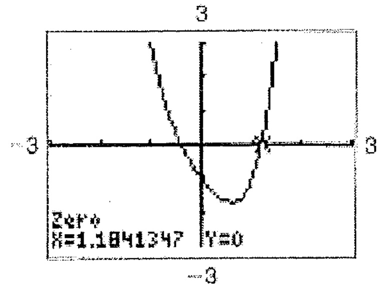


Figure 2.1-4b

Example 4 Solving $\dfrac{f(x)}{g(x)} = 0$

Solve $\dfrac{2x^2 + x - 1}{9x^2 - 9x + 2} = 0.$ → set numerator = 0

$Y_1 = 2x^2 + x - 1$

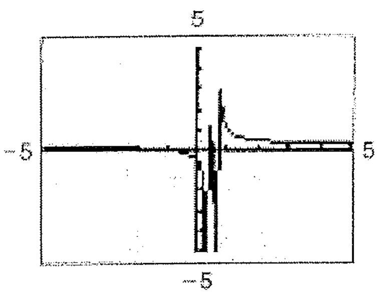


Figure 2.1-5

The graph of $y = \dfrac{2x^2 + x - 1}{9x^2 - 9x + 2}$ in Figure 2.1-5 is impossible to read. Using the zoom feature will display a better graph, but it may be easier to use the fact that a fraction is zero only when its numerator is zero and its denominator is nonzero.

The values that make the numerator zero can easily be found by finding the zeros of $y = 2x^2 + x - 1$. Discard any value that makes the denominator of the original equation zero because

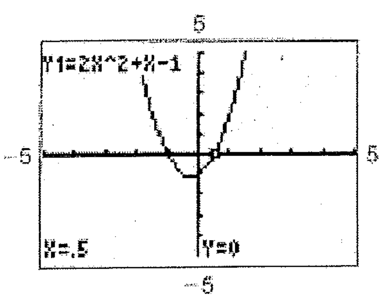


Figure 2.1-6

Figure 2.1-6 shows that one x-intercept of $y = 2x^2 + x - 1$ is $x = 0.5$ and the other is $x = -1$ (not identified on the graph). Neither value makes the denominator zero, so they are the solutions to the given equation, which can be verified by substitution.

How to check your solution…..

Clear screen, type equation, hit enter and you should get a zero!

Exercises 2.1

In Exercises 1–6, determine graphically the number of solutions of the equation, but don't solve the equation. You may need a viewing window other than the standard one to find all of the x-intercepts.

DO NOT SOLVE THESE ONES!!!

1. $x^5 + 5 = 3x^4 + x$

3 sol.

3. $x^7 - 10x^5 + 15x + 10 = 0$

3 sol.

5. $x^4 + 500x^2 - 8000x = 16x^3 - 32{,}000$

2 sol.

In Exercises 7–34, use a graphical method to find all real solutions of the equation, approximating when necessary.

7. $x^3 + 4x^2 + 10x + 15 = 0$

X= -2.426

9. $x^4 + x - 3 = 0$

X= -1.453
X= 1.164

11. $\sqrt{x^4 + x^3 - x - 3} = 0$

X= -1.475
X= 1.237

13. $\sqrt{\frac{2}{5}x^5 + x^2 - 2x} = 0$

X=0 X= 1.192
X= -1.750

15. $x^2 = \sqrt{x + 5}$

X= -1.379
X= 1.603

17. $\dfrac{2x^5 - 10x + 5}{x^3 + x^2 - 12x} = 0$

X= -1.601
X= .507
X= 1.329

19. $\dfrac{x^3 - 4x + 1}{x^2 + x - 6} = 0$

X= -2.115
X= .254
X= 1.861

21. $2x^3 - 4x^2 + x - 3 = 0$

X= 2.102

23. $x^5 - 6x + 6 = 0$

X= -1.752

25. $10x^5 - 3x^2 + x - 6 = 0$

X= .951

27. $2x - \frac{1}{2}x^2 - \frac{1}{12}x^4 = 0$

x= 0, X=2.207

29. $\dfrac{5x}{x^2 + 1} - 2x + 3 = 0$

$Y_1 = (5x/(x^2+1)) - 2x + 3$

x= 2.390

31. $|x^2 - 4| = 3x^2 - 2x + 1$

X= -.651
X= 1.151

33. $\sqrt{x^2 + 3} = \sqrt{x - 2} + 5$

$Y_1 = \sqrt{x^2+3} - \sqrt{x-2} - 5$

X= 7.033