

Algebra 2

Section 3-2 – Notes and Examples Solving Systems Algebraically

Name: Key
Date: _____ Hour: _____

Graphing is not always a great choice for solving systems. First, it can take a long time. Second, if the solution is a fraction, you can not always determine exactly what that solution truly is!

There are two other common methods, called *elimination* and *substitution*.

The Elimination Method

- ❶ Make sure both equations are in standard form and all variables and constants are “lined up.”
- ❷ Determine if the coefficients on either variable are opposites of each other. If so, go on to Step #4.
- ❸ Multiply one or both equations by a number (it can be a different number for each equation) so that coefficients on the same variable are opposites of each other. Be sure to multiply EVERYTHING in each equation by the value you choose for each equation.
- ❹ Add down the columns. One of the variables should cancel out.
- ❺ Solve the remaining equation.
- ❻ Use that answer to substitute into either of the two original equations to find the other value.

Example 2 – Use elimination to solve each system. **Classify** your result as before.

a.
$$\begin{cases} 2x - 3y = 14 \\ 4x + 3y = 46 \end{cases}$$

$$\begin{array}{r} 6x = 60 \\ x = 10 \end{array}$$

$$\begin{array}{r} 4(10) + 3y = 46 \\ 3y = 6 \\ y = 2 \end{array}$$

$$(10, 2)$$

b.
$$\begin{cases} 2x - 3y = 8 \\ x + 5y = -9 \end{cases}$$

$$\begin{array}{r} 2x - 3y = 8 \\ -2x - 10y = 18 \end{array}$$

$$\begin{array}{r} -13y = 26 \\ -13 \end{array}$$

$$y = -2$$

$$x + 5(-2) = -9$$

$$x - 10 = -9$$

$$x = 1$$

$$(1, -2)$$

c.
$$\begin{cases} 6x + 5y = 22 \\ 9x + 2y = 11 \end{cases}$$

$$\begin{array}{r} 12x + 10y = 44 \\ -45x - 10y = -55 \end{array}$$

$$-33x = -11$$

$$x = \frac{-11}{-33} = \frac{1}{3}$$

$$3\left(\frac{1}{3}\right) + 2y = 11$$

$$\begin{array}{r} 3 + 2y = 11 \\ -3 \end{array}$$

$$2y = 8$$

$$y = 4$$

$$\left(\frac{1}{3}, 4\right)$$

The Substitution Method

- ❶ Pick one equation that has a variable by itself. If there isn't one, pick the equation where it would be easiest to get a variable by itself (look for a variable with no coefficient).
- ❷ Re-write the OTHER equation using parentheses in place of the variable that you chose in Step #1.
- ❸ Substitute the first equation you chose into the second equation. The variable that was alone in the first equation should "disappear" and be replaced by an expression using the other variable.
- ❹ Solve this single equation in one variable.
- ❺ Use that answer to substitute back into the equation where a variable was alone to find the other value.

Example 1 – Use substitution to solve each system. **Classify** your result as before.

a.
$$\begin{cases} y = (2x - 1) \\ 4x - 6y = 10 \end{cases}$$

$$4x - 6(2x - 1) = 10$$

$$4x - 12x + 6 = 10$$

$$-8x = 4$$

$$x = -\frac{1}{2}$$

$$y = 2\left(-\frac{1}{2}\right) - 1$$

$$y = -1 - 1$$

$$y = -2 \quad \left(-\frac{1}{2}, -2\right)$$

b.
$$\begin{cases} x + 2y = 2 \rightarrow x = (-2y + 2) \\ 4x - 5y = -31 \end{cases}$$

$$4(-2y + 2) - 5y = -31$$

$$-8y + 8 - 5y = -31$$

$$-13y + \frac{8}{-8} = \frac{-31}{-8}$$

$$\frac{-13y}{-13} = \frac{-39}{-13}$$

$$y = 3$$

$$x = -4$$

$$(-4, 3)$$

You can still recognize parallel and coinciding lines when you use the substitution and elimination methods. Watch for final statements that are:

- Always true (coinciding lines)
- Always false (parallel lines)

Example 3 – Use either substitution or elimination to solve each system. **Classify** your result as before.

a.
$$\begin{cases} 6x + 4y = 2 \\ -2(3x + 2y = -1) \end{cases}$$

$$\begin{array}{r} 6x + 4y = 2 \\ -6x - 4y = 2 \\ \hline 0 = 4 \end{array}$$

$$0 = 4$$

No Sol.

Parallel

Inconsistent / Indep.

b.
$$\begin{cases} y = (2x - 5) \\ -8x + 4y = -20 \end{cases}$$

$$-8x + 4(2x - 5) = -20$$

$$-8x + 8x - 20 = -20$$

$$-20 = -20$$

Infinite Sol.

Dependent

Same Line

Consistent