Name ______________________________ Date ______________ Hour ____________

1.4 Lines

A **graph** is a set of points in a plane.

A **solution of an equation** in two variables, say x and y, is a pair of numbers such that the substitution of the first number for x and the second number for y produces a true statement. The **graph of an equation** in two variables is the set of points in a plane whose coordinates are solutions of the equation.

Slope

When you move from a point P to a point Q on a line, two numbers are involved, as illustrated in Figure 1.4-2.

- The vertical distance you move is called the **change in** y, which is sometimes denoted Δy and read "delta y."
- The horizontal distance you move is called the **change in** x, which is sometimes denoted Δx and read "delta x."

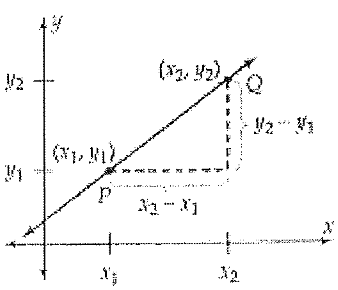


Figure 1.4-3

The fraction $\dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\Delta y}{\Delta x}$ measures the steepness of the line. Suppose P has coordinates (x_1, y_1) and Q has coordinates (x_2, y_2), as shown in Figure 1.4-3.

- The change in y is the difference of the y-coordinates of P and Q.

$$\Delta y = y_2 - y_1$$

- The change in x is the difference of the x-coordinates of P and Q.

$$\Delta x = x_2 - x_1$$

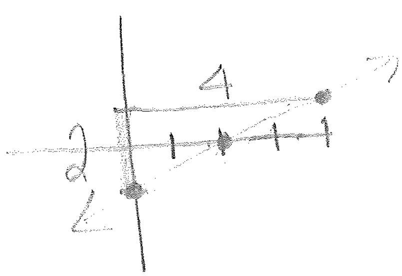
Slope of a Line

If (x_1, y_1) and (x_2, y_2) are points with $x_1 \neq x_2$, then the *slope* of the line through these points is the ratio

$$\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1 Finding Slope Given Two Points

Find the slope of the line that passes through $(0, -1)$ and $(4, 1)$.



$\frac{2}{4} = \frac{1}{2}$

$(0, -1)$
$(4, 1)$

$\dfrac{-1-1}{0-4}$

$\dfrac{-2}{-4} = \dfrac{1}{2}$

> **CAUTION**
>
> When finding slopes, you must subtract the y-coordinates and the x-coordinates in the same order. With the points (3, 4) and (1, 8), for instance, if you use $8 - 4$ in the numerator, you must write $1 - 3$ in the denominator, *not* $3 - 1$.

Properties of Slope

The slope of a nonvertical line is a number m that measures how steeply the line rises or falls.

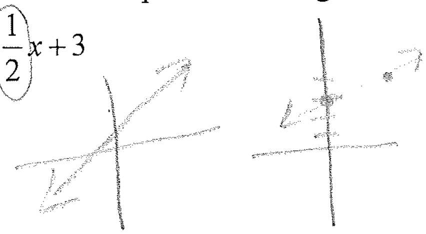
- If $m > 0$, the line rises from left to right; the larger m is, the more steeply the line rises. [*Lines L_1 and L_2*]
- If $m = 0$, the line is horizontal. [*Line L_3*]
- If $m < 0$, the line falls from left to right; the larger $|m|$ is, the more steeply the line falls. [*Lines L_4 and L_5*]

Which line is steeper, A or B? Take the absolute value if the slope and the larger number is the steeper sloped line.

A. $y = -2x + 3$

steeper

B. $y = \frac{1}{2}x + 3$



Slope-Intercept Form

The line with slope m and y-intercept b is the graph of the equation

$$y = mx + b.$$

Slope=m   y intercept=b

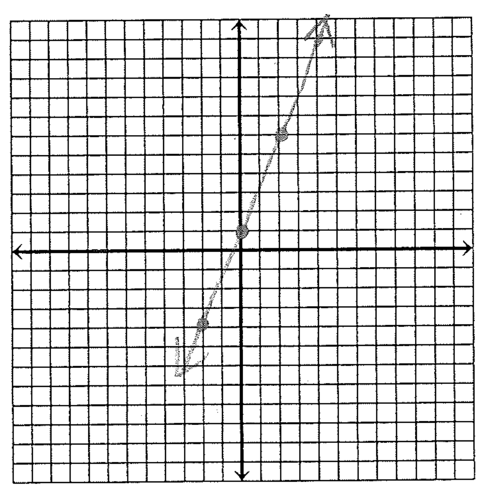
Example 4 Graphing a Line

Sketch the graph of $2y - 5x = 2$, an

$\frac{2y}{2} = \frac{5x}{2} + \frac{2}{2}$

$y = \frac{5}{2}x + 1$

b



Point-Slope Form

The line with slope m through the point (x_1, y_1) is the graph of the equation

$$\boxed{y - y_1 = m(x - x_1).}$$

Example 6 Point-Slope Form of a Line

Sketch the graph and find the equation of the line that passes through the point (1, −6) with slope 2. Write the equation in slope-intercept form.

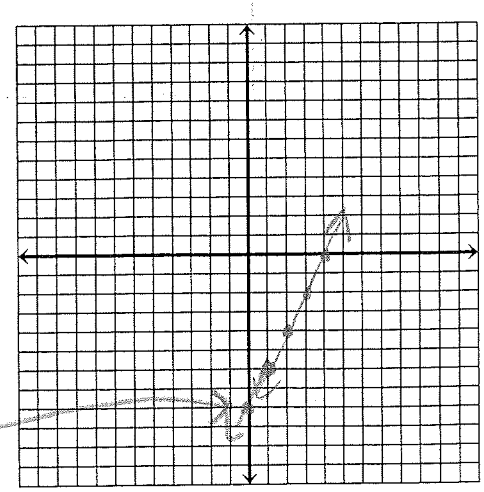
$y - y_1 = m(x - x_1)$

$y + 6 = 2(x - 1)$

$y + 6 = 2x - 2$

$y = mx + b$

$y = 2x - 8$

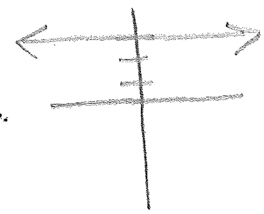


Vertical and Horizontal Lines

When a line has 0 slope, it is called a **horizontal line,** and it can be written as $y = 0x + b = b$.

Example 7 Equation of a Horizontal Line

Describe and sketch the graph of the equation $y = 3$.



Example 8 Equation of a Vertical Line

Find the equation of the vertical line shown in Figure 1.4-12.

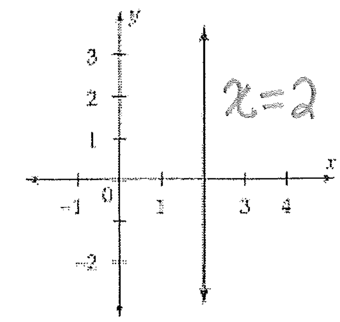


Figure 1.4-12

Parallel and Perpendicular Lines

The slope of a line measures how steeply it rises or falls. Because parallel lines rise or fall equally steeply, their slopes are the same.

Two lines that meet in a right angle, that is, a 90° angle, are said to be **perpendicular.** There is a close relationship between the slopes of two perpendicular lines.

Parallel and Perpendicular Lines

Two nonvertical lines are parallel when they have exactly the same slope.

Two nonvertical lines are perpendicular when the product of their slopes is −1.

Example 9 Parallel and Perpendicular Lines

Given the line M whose equation is $\boxed{3x - 2y + 6 = 0,}$ find the equation of the lines through the point $(2, -1)$

a. parallel to M.

b. perpendicular to M.

$$\frac{3x}{2}+\frac{6}{2}=\frac{2y}{2}$$

$$\frac{3}{2}x+3=y$$

$$\boxed{y=\frac{3}{2}x+3}$$

$y-y_1=m(x-x_1)$

a) $y+1=\frac{3}{2}(x-2)$

b) $y+1=-\frac{2}{3}(x-2)$

Standard Form of a Line

The *standard form of a line* is

$$Ax + By = C,$$

where A, B, and C are integers, $A \geq 0$, and A and B are not both 0.

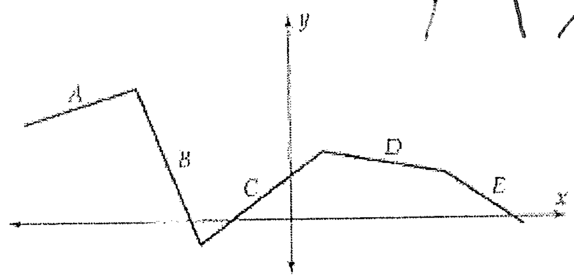
Rewrite the equation in slope intercept form: $2 \cdot \cancel{y=}\frac{1}{\cancel{2}}x+3 \cdot 2$

$2y = 1x + 6$

$$\boxed{\begin{array}{l} -1x+2y=6 \\ 1x-2y=-6 \end{array}}$$

✓

1. For which of the line segments in the figure is the slope
   a. largest? C
   b. smallest? B
   c. largest in absolute value? B
   d. closest to zero? D



In Exercises 17–20, find the equation of the line with slope m that passes through the given point.

17. $m = 1$; $(3, 5)$

$y - 5 = 1(x-3)$ point slope

$y = x + 2$ ← y intercept

$-x + y = 2$
$x - y = -2$ } standard form

In Exercises 3–6, find the slope and y-intercept of the line whose equation is given.

5. $3(x - 2) + y = 7 - 6(y + 4)$ $\quad m = -\frac{3}{7} \quad b = -\frac{11}{7}$

In Exercises 21–24, find the equation of the line through the given points.

21. $(0, -5)$ and $(-3, -2)$

$y + 5 = -1(x + 0)$
$y + 2 = -1(x + 3)$

$\underline{y = -x - 5}$

In Exercises 7–10, find the slope of the line through the given points.

9. $\left(\frac{1}{4}, 0\right); \left(\frac{3}{4}, 2\right)$ $\quad m = 4$

In Exercises 25–28, determine whether the line through P and Q is parallel or perpendicular to the line through R and S, or neither.

25. $P = (2, 5)$, $Q = (-1, -1)$ and $R = (4, 2)$, $S = (6, 1)$

$PQ = 2 \quad \frac{5+1}{2+1} = \frac{6}{3} = 2$

$RS = -\frac{1}{2}$

$\perp$

Perp.

In Exercises 11–14, find a number t such that the line passing through the two given points has slope -2.

13. $(t + 1, 5)$; $(6, -3t + 7)$ $\quad t = \frac{12}{5}$

$\frac{5 - (-3t + 7)}{(t+1) - 6} = \frac{-2}{1}$

$\frac{5 + 3t - 7}{t - 5} = \frac{-2}{1}$ $\quad 3t - 2 = -2t + 10$

$5t = 12$

$t = \frac{12}{5}$

$\frac{3t - 2}{t - 5} = \frac{-2}{1}$

In Exercises 29–31, determine whether the lines whose equations are given are parallel, perpendicular, or neither.

29. $2x + y - 2 = 0$ and $4x - 2y + 18 = 0$

$y = -2x + 2 \qquad \frac{2y}{2} = \frac{-4x - 18}{2}$

$m = -2 \qquad m = -2$

parallel

In Exercises 35–42, find an equation for the line satisfying the given conditions.

37. through (2, 3) and parallel to $3x - 2y = 5$

$\frac{3}{2}x - \frac{5}{2} = \frac{2}{2}y$

$y = \frac{3}{2}x$

$\boxed{y - 3 = \frac{3}{2}(x - 2)}$

$y - 3 = \frac{3}{2}x - 3$

$+3 \qquad +3$

$\boxed{y = \frac{3}{2}x}$

41. through (−1, 3) and perpendicular to the line through (0, 1) and (2, 3)

$m = 1 \Rightarrow m = -1$

$y - 3 = -1(x + 1)$

$y = -x + 2$

53. The poverty level income for a family of four was $9287 in 1981. Due to inflation and other factors, the poverty level income rose to approximately $18,267 in 2001. (Source: U.S. Census Bureau)

a. Find a linear equation that approximates the poverty level income y in year x (with $x = 0$ corresponding to 1981).

$m = 449$

$m = \frac{18,267 - 9287}{2001 - 1981}$

x	y
0	9287
20	18,267

$m = 449$

$y = 449x + 9287$

b. Use the equation of part a to estimate the poverty level income in 1990 and 2005.

$x = 9 \qquad x = 24$

1990 $x \to 9$ 13,328

2005 $x \to 24$ 20,063

52. Sales of a software company increased linearly from $120,000 in 1996 to $180,000 in 1999

$m = +$

a. Find an equation that expresses the sales y in year x (where $x = 0$ corresponds to 1996).

$y = 120,000 + 20,000x$

x	y
0 = 1996	120,000
1 = 1997	
2 = 1998	
3 = 1999	180,000
4 = 2000	

$m = \frac{60,000}{3} = 20,000$

$\boxed{y = 20,000x + 120,000}$

b. Estimate the sales in 2001.

$220,000

$x = 5 \to 2001$

$y = 20,000(5) + 120,000$

$= 220,000$

$y-$

61. A hat company has fixed costs of $50,000 and variable costs of $8.50 per hat.

a. Find an equation that gives the total cost y of producing x hats.

b. What is the average cost per hat when 20,000 are made? 50,000? 100,000?

$y = 50,000 + 8.50x$

$y = 8.50x + 50,000$

$y = 8.50(20,000) + 50,000$

$y = \frac{220,000}{20,000} = 11$

$y = 8.50(50,000) + 50,000$

$= \frac{475,000}{50,000} = 9.50$

$y = 8.50(100,000) + 50,000$

$= \frac{900,000}{100000} = 9$