Key

1.3 Arithmetic Sequences

Objectives

- Identify and graph an arithmetic sequence
- Find a common difference
- Write an arithmetic sequence recursively and explicitly
- Use summation notation
- Find the nth term and the nth partial sum of an arithmetic sequence

A sequence is a list of numbers in a certain order….

An infinite sequence is one where the same pattern continues for an infinite number of terms…… points of ellipsis

Finite= ends

Take note

Key Concept **Arithmetic Sequence**

An **arithmetic sequence** is a sequence where the difference between consecutive terms is constant. This difference is the **common difference**.

Problem 1 Identifying Arithmetic Sequences

Is the sequence an arithmetic sequence?

A 3, 6, 9, 12, 15, . . .

Find the differences between consecutive terms.

3 6 9 12 15

3 3 3 3

yes

B 1, 4, 9, 16, 25, . . .

Find the difference between consecutive terms.

1 4 9 16 25

3 5 7 9

not

Got It? 1. Is the sequence an arithmetic sequence?

a. 2, 4, 8, 16, . . .

NO

b. 1, 5, 9, 13, 17, . . .

$d=4$ yes

A recursive definition for this sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} + d$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a_1 + (n-1)d$, for $n \geq 1$

$a_n = a_1 + (n-1)d$

Examples like 1-5 …Write an explicit and recursive formula for each sequence.

A. 2, 4, 6, 8, 10…..

$+2 = d$

$a_1 = 2$

$a_n = a_{n-1} + 2$

$a_n = 2 + (n-1)2$

$a_n = 2 + 2n - 2$

$a_n = 2n$

B. 0, 6, 12, 18, 24…..

$d = 6$

$a_1 = 0$

$a_n = a_{n-1} + 6$

$a_n = 0 + (n-1)6$

$a_n = 6n - 6$

C. $a_1 = 2$ and d=3

$a_1 = 2$

$a_n = a_{n-1} + 3$

$a_n = \cancel{a_n}\, 2 + (n-1)3$

$2 + 3n - 3$

$a_n = 3n - 1$

Examples like 19-23.

A. Find the 45^{th} term of the arithmetic sequence 5, 9, 13…..

$a_n = a_1 + (n-1)d$

$a_{45} = 5 + (45-1)4$

$a_{45} = 181$

E. What is a_{18} if $a_5 = 27$ and d=5.

$a_n = a_1 + (n-1)d$

$27 = a_1 + (5-1)5$

$7 = a_1$

$a_{18} = 7 + (18-1)5$

$= 92$

$U_3 = 94$ $U_6 = 85$

Examples like 25-29…..

A. Write the nth term if $a_3 = 94$ and $a_6 = 85$.

$a_3 = 94$ $a_6 = 85$

$(3,94)$ $(6,85)$

$d = m = \frac{94-85}{3-6} = -3$

$a_n = a_1 + (n-1)d$

$94 = a_1 + (3-1)d$

B. Find the 45^{th} term of the arithmetic sequence given $a_6 = 57$ and $a_{10} = 93$

$(6,57)$ $(10,93)$

$d = \frac{93-57}{10-6} = \frac{36}{4} = 9$

$a_n = a_1 + (n-1)d$

$57 = a_1 + (6-1)(9)$

$12 = a_1$

$a_n = 12 + (n-1)9$

$a_{45} = 12 + (45-1)9$

$= 408$

Just as you found formulas for terms of sequences, you can find formulas for the sums of the terms of sequences.

Essential Understanding When you know two terms and the number of terms in a finite arithmetic sequence, you can find the sum of the terms.

A **series** is the indicated sum of the terms of a sequence. A **finite series**, like a finite sequence, has a first term and a last term, while an **infinite series** continues without end.

Finite sequence	**Finite series**
6, 9, 12, 15, 18	6 + 9 + 12 + 15 + 18 (The sum is 60.)
Infinite sequence	**Infinite series**
3, 7, 11, 15, . . .	3 + 7 + 11 + 15 + . . .

$$\sum_{n=1}^{k} u_n = \frac{k}{2}(u_1 + u_n)$$

An **arithmetic series** is a series whose terms form an arithmetic sequence (as shown above). When a series has a finite number of terms, you can use a formula involving the first and last term to evaluate the sum.

Take note

Property Sum of a Finite Arithmetic Series

The sum S_n of a finite arithmetic series $a_1 + a_2 + a_3 + \cdots + a_n$ is

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where a_1 is the first term, a_n is the nth term, and n is the number of terms.

$36-15=21+1$

Examples like 7-11, 31,33

$K=22$

A. Find the 12th partial sum of

-8, -3, 2,7…..

$U_{12}=-8+(12-1)5$

$U_{12}=47$

$\frac{K}{2}(U_1+U_K)$

$\frac{12}{2}(-8+47)=234$

B. $\sum_{n=1}^{50} 7-3n$

$K=50$

$\frac{K}{2}(U_1+U_K)$

$\frac{50}{2}(4+-143)$

-3475

C. $\sum_{n=15}^{36} 2n-8$

$\frac{22}{2}(22+64)$

946

$72-8=64$

$30-8=22$

Problem 1 Finding the Sum of a Finite Arithmetic Series

What is the sum of the even integers from 2 to 100?

The series $2+4+6+\cdots+100$ is arithmetic with first term 2, last (and 50th) term 100, and common difference 2. The sum is

$$S_{50}=\frac{50}{2}(2+100)=25(102)=2550.$$

Got It? 1. a. What is the sum of the finite arithmetic series $4+9+14+19+24+\cdots+99$?

$d=5$

$\frac{K}{2}(U_1+U_K)$

$\frac{20}{2}(4+99)=1030$

How to find K

$99=4+(n-1)5$

$95=5n-5$

$100=5n$

$20=n$

Problem 2 Using the Sum of a Finite Arithmetic Series

Bonus A company pays a \$10,000 bonus to salespeople at the end of their first 50 weeks if they make 10 sales in their first week, and then improve their sales numbers by two each week thereafter. One salesperson qualified for the bonus with the minimum possible number of sales. How many sales did the salesperson make in week 50? In all 50 weeks?

$U_{50}=$ 50th week

$U_{50}=10+(50-1)2$

$10+(49)2$

10, 12, 14, 16, 18….

$49 \times 2 = 98$

$U_{50}=108$

$\frac{K}{2}(U_1+U_K)=\frac{50}{2}(10+108)$

$25(118)=2950$