

Key

Objectives

- Identify and graph an arithmetic sequence
- Find a common difference
- Write an arithmetic sequence recursively and explicitly
- Use summation notation
- Find the n th term and the n th partial sum of an arithmetic sequence

A sequence is a list of numbers in a certain order...

An infinite sequence is one where the same pattern continues for an infinite number of terms..... points of ellipsis

Finite= ends



Key Concept Arithmetic Sequence

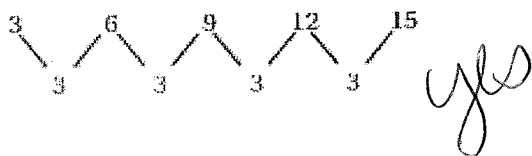
An arithmetic sequence is a sequence where the difference between consecutive terms is constant. This difference is the common difference.

Problem 1 Identifying Arithmetic Sequences

Is the sequence an arithmetic sequence?

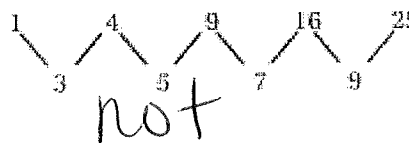
A 3, 6, 9, 12, 15, ...

Find the differences between consecutive terms.



B 1, 4, 9, 16, 25, ...

Find the difference between consecutive terms.



Got It? 1. Is the sequence an arithmetic sequence?

a. 2, 4, 8, 16, ...

NO

b. 1, 5, 9, 13, 17, ...

$d=4$ yes

A recursive definition for this sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} + d$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a_1 + (n-1)d$, for $n \geq 1$

$a_n = a_1 + (n-1)d$

Examples like 1-5 ... Write an explicit and recursive formula for each sequence.

A. $\rightarrow \rightarrow +2=d$
2, 4, 6, 8, 10, ...

$a_1 = 2$
 $a_n = a_{n-1} + 2$

$a_n = 2 + (n-1)2$
 $a_n = 2 + 2n - 2$
 $a_n = 2n$

B. $\rightarrow \rightarrow \rightarrow d=6$
0, 6, 12, 18, 24, ...

$a_1 = 0$
 $a_n = a_{n-1} + 6$

$a_n = 0 + (n-1)6$
 $a_n = 6n - 6$

C. $a_1 = 2$ and $d=3$

$a_1 = 2$
 $a_n = a_{n-1} + 3$

$a_n = 2 + (n-1)3$
 $2 + 3n - 3$
 $a_n = 3n - 1$

Examples like 19-23.

A. Find the 45th term of the arithmetic sequence 5, 9, 13, ...

$$a_n = a_1 + (n-1)d$$

$$a_{45} = 5 + (45-1)4$$

$$a_{45} = 181$$

E. What is a_{18} if $a_5 = 27$ and $d=5$.

$$a_n = a_1 + (n-1)d$$

$$27 = a_1 + (5-1)5$$

$$7 = a_1$$

$$a_{18} = 7 + (18-1)5$$

$$= 92$$

$$u_3 = 94 \quad u_6 = 85$$

Examples like 25-29, ...

A. Write the nth term if $a_3 = 94$ and $a_6 = 85$.

$$a_3 = 94 \quad a_6 = 85$$

$$(3, 94) \quad (6, 85)$$

$$d = m = \frac{94 - 85}{3 - 6} = -3$$

$$a_n = a_1 + (n-1)d$$

$$94 = a_1 + (3-1)d$$

B. Find the 45th term of the arithmetic sequence given

$$a_6 = 57 \text{ and } a_{10} = 93$$

$$(6, 57) \quad (10, 93)$$

$$d = \frac{93 - 57}{10 - 6} = \frac{36}{4} = 9$$

$$a_n = a_1 + (n-1)d$$

$$57 = a_1 + (6-1)(9)$$

$$12 = a_1$$

$$a_n = 12 + (n-1)9$$

$$a_{45} = 12 + (45-1)9$$

$$= 408$$

Just as you found formulas for terms of sequences, you can find formulas for the sums of the terms of sequences.

Essential Understanding When you know two terms and the number of terms in a finite arithmetic sequence, you can find the sum of the terms.

A **series** is the indicated sum of the terms of a sequence. A **finite series**, like a finite sequence, has a first term and a last term, while an **infinite series** continues without end.

Finite sequence

6, 9, 12, 15, 18

Infinite sequence

3, 7, 11, 15, ...

Finite series

6 + 9 + 12 + 15 + 18 (The sum is 60.)

Infinite series

3 + 7 + 11 + 15 + ...

$$\sum_{n=1}^k u_n = \frac{k}{2}(u_1 + u_n)$$

An **arithmetic series** is a series whose terms form an arithmetic sequence (as shown above). When a series has a finite number of terms, you can use a formula involving the first and last term to evaluate the sum.



Property Sum of a Finite Arithmetic Series

The sum S_n of a finite arithmetic series $a_1 + a_2 + a_3 + \dots + a_n$ is

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where a_1 is the first term, a_n is the n th term, and n is the number of terms.

Examples like 7-11, 31,33

$$36 - 15 = 21 + 1$$

$$k = 22$$

A. Find the 12th partial sum of

-8, -3, 2, 7, ...

$$U_{12} = -8 + (12-1)5$$

$$U_{12} = 47$$

$$\frac{k}{2}(u_1 + u_k)$$

$$\frac{12}{2}(-8 + 47) = 234$$

B. $\sum_{n=1}^{50} 7 - 3n$

$$k = 50$$

$$\frac{k}{2}(u_1 + u_k)$$

$$\frac{50}{2}(4 + -143)$$

$$-3475$$

C. $\sum_{n=15}^{36} 2n - 8$

$$\frac{22}{2}(22 + 64)$$

$$946$$

$$\begin{array}{r} 72 \\ -8 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 30 \\ -8 \\ \hline 22 \end{array}$$

Problem 1 Finding the Sum of a Finite Arithmetic Series

What is the sum of the even integers from 2 to 100?

The series $2 + 4 + 6 + \dots + 100$ is arithmetic with first term 2, last (and 50th) term 100, and common difference 2. The sum is

$$S_{50} = \frac{50}{2}(2 + 100) = 25(102) = 2550.$$

Got It? 1. a. What is the sum of the finite arithmetic series

$4 + 9 + 14 + 19 + 24 + \dots + 99$

$$d = 5$$

$$\frac{k}{2}(u_1 + u_k) = 1030$$

$$\frac{20}{2}(4 + 99)$$

How to find k

$$99 = 4 + (n-1)5$$

$$95 = 5n - 5$$

$$100 = 5n$$

$$20 = n$$

Problem 2 Using the Sum of a Finite Arithmetic Series

Bonus A company pays a \$10,000 bonus to salespeople at the end of their first 50 weeks if they make 10 sales in their first week, and then improve their sales numbers by two each week thereafter. One salesperson qualified for the bonus with the minimum possible number of sales. How many sales did the salesperson make in week 50? In all 50 weeks?

$$U_{50} = 50^{\text{th}} \text{ week}$$

$$U_{50} = 10 + (50-1)2$$

$$10 + (49)2$$

10, 12, 14, 16, 18, ...

$$\begin{array}{r} 49 \\ \times 2 \\ \hline 98 \end{array}$$

$$U_{50} = 108$$

$$\frac{k}{2}(u_1 + u_k) = \frac{50}{2}(10 + 108)$$

$$25(118) = 2950$$

