1.6 Geometric Sequences

Content Standard

Prepares for A.SSE.4 Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems.

Objective To define, identify, and apply geometric sequences

You build a *geometric sequence* by multiplying each term by a constant.

Essential Understanding In a *geometric sequence*, the ratio of any term to its preceding term is a constant value.

take note

Key Concept Geometric Sequence

A **geometric sequence** with a starting value a and a **common ratio** r is a sequence of the form

$$a, ar, ar^2, ar^3, \ldots$$

A recursive definition for the sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} \cdot r$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

Examples like 1-8….

Are the following arithmetic, geometric or neither?

A. 3, 9,27, 81……

geometric
$r = 3$

recursive
$a_1 = 3$
$a_n = a_{n-1} \cdot r$

explicit
$a_n = a_1 \cdot r^{n-1}$
$a_n = 3 \cdot 3^{n-1}$

B. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8} \ldots$

$\frac{5}{4} - \frac{5 \cdot 2}{2 \cdot 2} = -\frac{5}{4}$

$\frac{5}{8} - \frac{5 \cdot 2}{4 \cdot 2} = -\frac{5}{8}$

not arithmetic

geometrc
$r = 1/2$

C. 1.5, 4.5, 13….

$4.5 - 1.5 = 3$
$13.0 - 4.5 =$
Not arith

$\frac{4.5}{1.5} = 3$

$\frac{13}{4.5} = 2.8$

not geo

Neither!

$$recursive\begin{Bmatrix} u_1 = \_\_\_ \\ u_n = ru_{n-1} \end{Bmatrix} \qquad explicit\left\{u_n = u_1 r^{n-1}\right\}$$

Write the recursive and explicit formulas and find the 6th term….

$r = -\frac{1}{5}$

Like 9-14. A. Given $2, -\frac{2}{5}, \frac{2}{25}, \ldots$

$a_1 = 2$

$a_n = a_{n-1}\left(-\frac{1}{5}\right)$

$a_n = 2\cdot\left(-\frac{1}{5}\right)^{n-1}$

$a_6 = 2\left(-\frac{1}{5}\right)^{6-1}$

$a_6 = 2\left(-\frac{1}{5}\right)^5$

$= \frac{2}{1}\left(\frac{-1}{3125}\right)$

$= \frac{-2}{3125}$

Find the 10th term of the geometric sequence 4, 12, 36, __, __, __, __, __, __, __

$r = 3$ $\qquad u_{10}$

$$\frac{a_1(1-r^n)}{1-r}$$

$$\frac{4(1-3^{10})}{1-3} = 78{,}732$$

Find the 2nd and 3rd term of the sequence 2, ____, ____, -54, …

Essential Understanding Just as with finite arithmetic series, you can find the sum of a finite geometric series using a formula. You need to know the first term, the number of terms, and the common ratio.

A **geometric series** is the sum of the terms of a geometric sequence.

Take note

Key Concept **Sum of a Finite Geometric Series**

The sum S_n of a finite geometric series $a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1}, r \neq 1$, is

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

where a_1 is the first term, r is the common ratio, and n is the number of terms.

Like 15-18, and 29-33.

Find the k^{th} partial sum of the geometric sequence u_n, with a common ratio r.

16. k=8, $u_1 = 9$, $r = \frac{1}{3}$

$9 + 3 + 1$

$u_1, u_2, u_3 \qquad u_8$

$$S_8 = \frac{a_1(1-r^n)}{1-r}$$

$$\frac{9(1-\frac{1}{3}^8)}{1-\frac{1}{3}}$$

Find the sum of the first 8 terms in the sequence… $\frac{-3}{2}, \frac{3}{4}, \frac{-3}{8}, \ldots$

$r = -\frac{1}{2}$

$$\frac{a_1(1-r^n)}{1-r}$$

$$\frac{-\frac{3}{2}(1-(-\frac{1}{2})^8)}{1-(-\frac{1}{2})} = \frac{-255}{256} \text{ or } -.996$$

Like 29-33. $\sum_{n=1}^{6} 3\left(\frac{1}{3}\right)^{n-1}$

u_1 $3\left(\frac{1}{3}\right)^{1-1}$ 3

u_2 $3\left(\frac{1}{3}\right)^{2-1}$ 1

$r = \frac{1}{3}$

$4\frac{40}{81}$

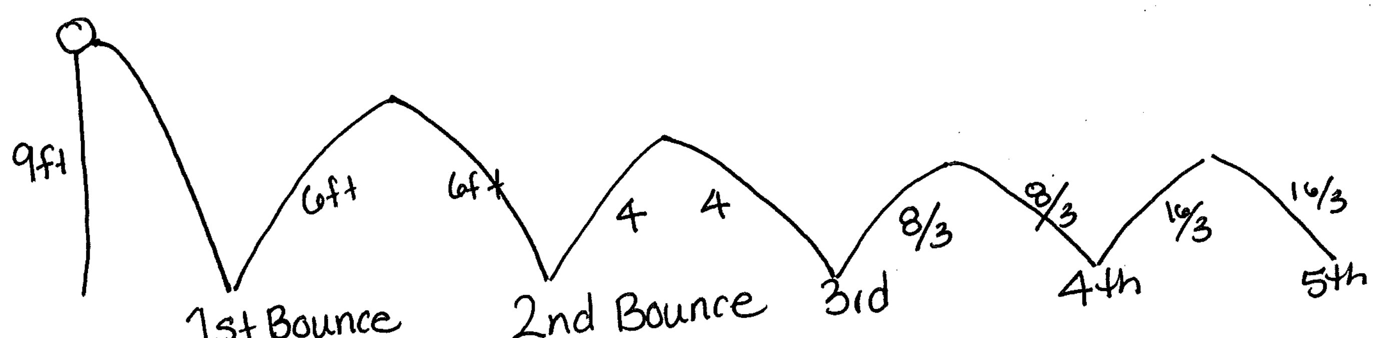
$$\frac{a_1(1-r^n)}{1-r}$$

$$\frac{3(1-\frac{1}{3}^6)}{1-\frac{1}{3}}$$

$$\frac{364}{81}$$

A ball is dropped from a height of 9 feet. It bounces 2/3 of its' previous height each time. Write explicit and recursive formulas for this problem. Determine the height after the 4th bounce and determine the total feet on the 5th bounce.

$\overset{3}{\cancel{9}} \cdot \frac{2}{\cancel{3}} = 6$ $\quad \overset{2}{\cancel{6}} \cdot \frac{2}{\cancel{3}} = 4$ $\quad 4 \cdot \frac{2}{3} = \frac{8}{3}$ $\quad \frac{8}{3} \cdot \frac{2}{3}$ $\frac{16}{3}$



$12, \quad 8, \quad \frac{16}{3}, \quad \frac{32}{3},$

$$\frac{a_1(1-r^n)}{1-r}$$

$$\frac{12(1-\frac{2}{3}^4)}{1-2/3} \approx 28.88 \text{ ft}$$

28.88
$+\ 9$
37.88 ft